HW #1:
Section 5.1: 2~56 (even)
Section 5.3: 2~62 (even)
Section 5.4: 2~64 (even)

* All the answers to the even numbered questions are erased, but the answers to the odd questions are there so that you can use them to see if you are getting the right answers.

FOR FURTHER THOUGHT

Prime Factor Splicing

Prime factor splicing (PFS) has received increasing attention in recent years. A good basic explanation can be found in the article "Home Primes and Their Families," in *Mathematics Teacher*, vol. 107, no. 8, April 2014. Like many topics in number theory, PFS concerns easily stated problems but quickly leads to difficult and unanswered questions. Here is the process:

- Begin with a composite natural number, say 15.
- Obtain its prime factorization: $15 = 3 \cdot 5$
- Form a new natural number by arranging all the prime factors, in ascending order: 35
- Repeat the steps until a prime number is produced, called the **home prime**. For 15, we can summarize the entire process as shown here.

$$15 = 3 \cdot 5 \rightarrow 35 = 5 \cdot 7 \rightarrow 57 = 3 \cdot 19 \rightarrow 319 = 11 \cdot 29 \rightarrow 1129 \text{ (a prime)}$$

If we call the prime 1129 the "parent," then the composite 15 is its "child." If two or more children are found to have the same parent, call them "siblings." The Sieve of Eratosthenes (see **page 179**) provides plenty of composites to get started. We consider just a few arbitrarily chosen examples here. To do very much prime factor splicing, you would want to use some computer algebra system (CAS) for the factoring. In the case above, the child is 15, the parent is 1129, and the four arrows show that it took four iterations to get from child to parent.

For Group or Individual Investigation

Complete the table, and then answer the questions that follow it.

Child	Number of Iterations	Parent
4	______	______
6	______	______
9	______	______
10	______	______
12	______	______
22	______	______
25	______	______
33	______	______
46	______	______
55	______	______

Questions

- Is it possible for a child to have more than one parent?
- Can a parent have more than one child?
- It is possible for PFS to require at least ____ iterations.
- It is possible for a family to include at least _____ siblings.
- Show that if 49 and 77 both have parents, they must be the same.

Interesting Facts

- The child 80 requires 31 iterations and has a 48-digit prime parent.
- 49 and 77 have been "spliced" to at least 110 and 109 iterations, respectively, and no one knows whether a parent exists for them.

The icon indicates that the answer is located in the "Answers to Selected Exercises" section at the end of the text.

5.1 EXERCISES

Decide whether each statement is true *or* false.

1. If n is a natural number and $9 \mid n$, then $3 \mid n$. true
2. If n is a natural number and $5 \mid n$, then $10 \mid n$.
3. Every natural number is divisible by 1. true
4. There are no even prime numbers.
5. 1 is the least prime number. false
6. Every natural number is both a factor and a multiple of itself.
7. If 16 divides a natural number, then 2, 4, and 8 must also divide that natural number. true
8. The prime number 53 has exactly two natural number factors.

Find all natural number factors of each number.

9. 12 1, 2, 3, 4, 6, 12

10. 20

11. 28 1, 2, 4, 7, 14, 28

12. 172

Use divisibility tests to decide whether the given number is divisible by each number.

(a) 2 **(b)** 3 **(c)** 4 **(d)** 5 **(e)** 6 **(f)** 8
(g) 9 **(h)** 10 **(i)** 12

13. 321

14. 540

15. 36,360

16. 123,456,789

17. (a) In constructing the Sieve of Eratosthenes for 2 through 100, we said that any composite in that range had to be a multiple of some prime less than or equal to 7 (since the next prime, 11, is greater than the square root of 100). Explain. Answers will vary.

(b) To extend the Sieve of Eratosthenes to 200, what is the largest prime whose multiples would have to be considered? 13

(c) Complete this statement: In seeking prime factors of a given number, we need only consider all primes up to and including the square root of that number, since a prime factor greater than the square root can occur only if there is at least one other prime factor less than the square root.

(d) Complete this statement: If no prime less than or equal to $\sqrt{n}$ divides n, then n is a prime number.

18. (a) Continue the Sieve of Eratosthenes in **Table 1** from 101 to 200 and list the primes between 100 and 200. How many are there?

(b) From your list in part (a), verify that the numbers 197 and 199 are both prime.

19. In your list for **Exercise 18(a),** consider the six largest primes less than 200. Which pairs of these would have products that end in the digit 7?
179 and 193, 181 and 197, 191 and 197, 193 and 199

20. By checking your pairs of primes from **Exercise 19,** give the prime factorization of 35,657.

21. List two primes that are consecutive natural numbers. Can there be any others? 2, 3; no

22. Can there be three primes that are consecutive natural numbers? Explain.

23. For a natural number to be divisible by both 2 and 5, what must be true about its last digit? It must be 0.

24. Consider the divisibility tests for 2, 4, and 8 (all powers of 2). Use inductive reasoning to predict the divisibility test for 16. Then use the test to show that 456,882,320 is divisible by 16.

25. Redraw the factor tree of **Example 5,** assuming that you first observe that $1320 = 12 \cdot 110$, then that $12 = 3 \cdot 4$ and $110 = 10 \cdot 11$. Complete the process and give the resulting prime factorization.

26. Explain how your result in **Exercise 25** illustrates the fundamental theorem of arithmetic.

Find the prime factorization of each composite number.

27. 126 $2 \cdot 3^2 \cdot 7$

28. 825

29. 1183 $7 \cdot 13^2$

30. 340

Here is a divisibility test for 7.

(a) *Double the last digit of the given number, and subtract this value from the given number with the last digit omitted.*

(b) *Repeat the process of part (a) as many times as necessary until it is clear whether the number obtained is divisible by 7.*

(c) *If the final number obtained is divisible by 7, then the given number also is divisible by 7. If the final number is not divisible by 7, then neither is the given number.*

Use this divisibility test to determine whether each number is divisible by 7.

31. 226,233 yes

32. 548,184

33. 496,312 no

34. 368,597

Here is a divisibility test for 11.

(a) *Starting at the left of the given number, add together every other digit.*

(b) *Add together the remaining digits.*

(c) *Subtract the smaller of the two sums from the larger. (If they are the same, the difference is 0.)*

(d) *If the final number obtained is divisible by 11, then the given number also is divisible by 11. If the final number is not divisible by 11, then neither is the given number.*

Use this divisibility test to determine whether each number is divisible by 11.

35. 6,524,846 no

36. 108,410,313

37. 60,128,459,358 yes

38. 29,630,419,088

39. Consider the divisibility test for the composite number 6, and make a conjecture for the divisibility test for the composite number 15.

40. Give two factorizations of the number 75 that are not prime factorizations.

Determine all possible digit replacements for x so that the first number is divisible by the second. For example, 37,58x is divisible by 2 if

$$x = 0, 2, 4, 6, \text{ or } 8.$$

41. 398,87x; 2 0, 2, 4, 6, 8

42. 2,45x,765; 3

43. 64,537,84x; 4 0, 4, 8

44. 2,143,89x; 5

45. 985,23x; 6 0, 6

46. 23, x54,470; 10

There is a method to determine the ***number of divisors*** *of a composite number. To do this, write the composite number in its prime factored form, using exponents. Add 1 to each exponent and multiply these numbers. Their product gives the number of divisors of the composite number. For example,*

$$24 = 2^3 \cdot 3 = 2^3 \cdot 3^1.$$

Now add 1 to each exponent:

$$3 + 1 = 4, \quad 1 + 1 = 2.$$

Multiply 4 · 2 to get 8. There are 8 divisors of 24. (Because 24 is rather small, this can be verified easily. The divisors are 1, 2, 3, 4, 6, 8, 12, and 24—a total of eight, as predicted.)

Find the number of divisors of each composite number.

47. 105 8

48. 156

49. $5^8 \cdot 29^2$ 27

50. $2^4 \cdot 7^2 \cdot 13^3$

Leap years occur when the year number is divisible by 4. An exception to this occurs when the year number is divisible by 100 (that is, it ends in two zeros). In such a case, the number must be divisible by 400 in order for the year to be a leap year. Determine which years are leap years.

51. 1556 leap year

52. 1990

53. 2200 not a leap year

54. 2400

55. Why is the following *not* a valid divisibility test for the number 8? "A number is divisible by 8 if it is divisible by both 4 and 2." Support your answer with an appropriate example. Answers will vary.

56. Choose any three consecutive natural numbers, multiply them together, and divide the product by 6. Repeat this several times, using different choices of three consecutive numbers. Make a conjecture concerning the result.

57. Explain why the product of three consecutive natural numbers must be divisible by 6. Include examples in your explanation. Answers will vary.

58. Choose any 6-digit number consisting of three digits followed by the same three digits in the same order (for example, 467,467). Divide by 13. Divide by 11. Divide by 7. What do you notice? Why do you think this happens?

One of the authors has three sons who were born, from eldest to youngest, on August 30, August 31, and October 14. For most (but not all) of each year, their ages are spaced two years from the eldest to the middle and three years from the middle to the youngest. In 2011, their ages were three consecutive prime numbers for a period of exactly 44 days. This same situation had also occurred in exactly two previous years. Use this information for ***Exercises 59–62****. (Hint: Consult the Sieve of Eratosthenes on* ***page 179*** *for a listing of all primes less than 100.)*

59. Which were the "two previous years" referred to above?
1981 and 1987

60. What were the years of birth of the three sons?

61. In what year will the same situation next occur? 2041

62. What will be the ages of the three sons at that time?

5.2 LARGE PRIME NUMBERS

OBJECTIVES

1 Understand the infinitude of primes.

2 Investigate several categories of prime numbers.

3 Learn how large primes are identified.

The Infinitude of Primes

One important basic result about prime numbers was proved by Euclid around 300 B.C., namely that there are infinitely many primes. This means that no matter how large a prime we identify, there are always others even larger. Euclid's proof remains today as one of the most elegant proofs in all of mathematics. (An *elegant* mathematical proof is one that demonstrates the desired result in a most direct, concise manner. Mathematicians strive for elegance in their proofs.) It is called a **proof by contradiction.**

A statement can be proved by contradiction as follows: Assume that the negation of the statement is true, and use that assumption to produce some sort of contradiction, or absurdity. Logically, the fact that the negation of the original statement leads to a contradiction means that the original statement must be true.

EXAMPLE 5 Applying a Theorem Proved by Fermat

One of the theorems legitimately proved by Fermat is as follows:

Every odd prime can be expressed as the difference of two squares in one and only one way.

Express each odd prime as the difference of two squares.

(a) 3 **(b)** 7

Solution

(a) $3 = 4 - 1 = 2^2 - 1^2$ **(b)** $7 = 16 - 9 = 4^2 - 3^2$

FOR FURTHER THOUGHT

Curious and Interesting

One of the most remarkable books on number theory is *The Penguin Dictionary of Curious and Interesting Numbers* (1986) by David Wells. This book contains fascinating numbers and their properties, including the following.

- There are only three sets of three digits that form prime numbers in all possible arrangements: {1, 1, 3}, {1, 9, 9}, {3, 3, 7}.
- Find the sum of the cubes of the digits of 136:

  $$1^3 + 3^3 + 6^3 = 244.$$

  Repeat the process with the digits of 244:

  $$2^3 + 4^3 + 4^3 = 136.$$

  We're back to where we started.
- 635,318,657 is the least number that can be expressed as the sum of two fourth powers in two ways:

  $$635{,}318{,}657 = 59^4 + 158^4 = 133^4 + 134^4.$$

- The number 24,678,050 has an interesting property:

  $$24{,}678{,}050 = 2^8 + 4^8 + 6^8 + 7^8 + 8^8 + 0^8 + 5^8 + 0^8.$$

- The number 54,748 has a similar interesting property:

  $$54{,}748 = 5^5 + 4^5 + 7^5 + 4^5 + 8^5.$$

- The number 3435 has this property:

  $$3435 = 3^3 + 4^4 + 3^3 + 5^5.$$

For anyone whose curiosity is piqued by such facts, the book mentioned above is for you!

For Group or Individual Investigation

Have each student in the class choose a three-digit number that is a multiple of 3. Add the cubes of the digits. Repeat the process until the same number is obtained over and over. Then, have the students compare their results. What is curious and interesting about this process?

5.3 EXERCISES

In Exercises 1–10 decide whether each statement is true *or* false.

1. Given a prime number, no matter how large, there is always another prime even larger. true
2. The prime numbers 2 and 3 are twin primes.
3. The first and third perfect numbers both end in the digit 6, and the second and fourth perfect numbers both end in the digits 28. true
4. For every Mersenne prime, there is a corresponding perfect number.
5. All prime numbers are deficient. true
6. The equation $17 + 51 = 68$ verifies Goldbach's conjecture for the number 68.
7. Even perfect numbers are more plentiful than Mersenne primes. false
8. The twin prime conjecture was proved in 2013.
9. The number $2^5(2^6 - 1)$ is perfect. false
10. Every natural number greater than 1 must be one of the following: prime, abundant, or deficient.
11. The proper divisors of 496 are 1, 2, 4, 8, 16, 31, 62, 124, and 248. Use this information to verify that 496 is perfect.

12. The proper divisors of 8128 are 1, 2, 4, 8, 16, 32, 64, 127, 254, 508, 1016, 2032, and 4064. Use this information to verify that 8128 is perfect.

13. As mentioned in the text, when $2^n - 1$ is prime,

$$2^{n-1}(2^n - 1)$$

is perfect. By letting $n = 2, 3, 5,$ and 7, we obtain the first four perfect numbers. Show that $2^n - 1$ is prime for $n = 13$, and then find the decimal digit representation for the fifth perfect number. 8191 is prime; 33,550,336

14. In the summer of 2014, the largest known prime number was $2^{57,885,161} - 1$. Use the formula in **Exercise 13** to write an expression for the perfect number generated by this prime number.

15. It has been proved that the reciprocals of *all* the positive divisors of a perfect number have a sum of 2. Verify this for the perfect number 28. $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{7} + \frac{1}{14} + \frac{1}{28} = 2$

16. Consider the following equations.

$$6 = 1 + 2 + 3$$
$$28 = 1 + 2 + 3 + 4 + 5 + 6 + 7$$

Show that a similar equation is valid for the third perfect number, 496.

Determine whether each number is abundant *or* deficient.

17. 32 deficient **18.** 60 **19.** 84 abundant **20.** 75

21. There are four abundant numbers between 1 and 25. Find them. (*Hint:* They are all even, and no prime number is abundant.) 12, 18, 20, 24

22. Explain why a prime number must be deficient.

23. There are no odd abundant numbers less than 945. If 945 has proper divisors 1, 3, 5, 7, 9, 15, 21, 27, 35, 45, 63, 105, 135, 189, and 315, determine whether it is abundant or deficient. $1 + 3 + 5 + 7 + 9 + 15 + 21 + 27 + 35 + 45 + 63 + 105 + 135 + 189 + 315 = 975$, and $975 > 945$, so 945 is abundant.

24. Explain in your own words the terms *perfect number, abundant number,* and *deficient number.*

25. The proper divisors of 1184 are 1, 2, 4, 8, 16, 32, 37, 74, 148, 296, and 592. The proper divisors of 1210 are 1, 2, 5, 10, 11, 22, 55, 110, 121, 242, and 605. Verify that 16-year-old Nicolo Paganini was correct about these two numbers in 1866. See the subsection "Amicable (Friendly) Numbers."

26. An Arabian mathematician of the ninth century stated the following: "If the three numbers

$$x = 3 \cdot 2^{n-1} - 1,$$
$$y = 3 \cdot 2^n - 1,$$

and

$$z = 9 \cdot 2^{2n-1} - 1$$

are all prime and $n \geq 2$, then $2^n xy$ and $2^n z$ are amicable numbers."

(a) Use $n = 2$, and show that the result is the least pair of amicable numbers, namely 220 and 284.

(b) Use $n = 4$ to obtain another pair of amicable numbers.

Write each even number as the sum of two primes. (There may be more than one way to do this.)

27. 12 5 + 7 **28.** 24 **29.** 40 11 + 29 **30.** 54

31. Joseph Louis Lagrange (1736–1813) conjectured that every odd natural number greater than 5 can be written as a sum $a + 2b$, where a and b are both primes.

(a) Verify this for the odd natural number 11.

(b) Verify that the odd natural number 17 can be written in this form in four different ways.

32. Another unproved conjecture in number theory states that every natural number multiple of 6 can be written as the difference of two primes. Verify this for 12 and 24.

Find one pair of twin primes between the two numbers given.

33. 45, 65 59 and 61 **34.** 65, 85

Pierre de Fermat provided proofs of many theorems in number theory. Exercises 35–38 investigate some of these theorems.

35. If p is prime and the natural numbers a and p have no common factor except 1, then $a^{p-1} - 1$ is divisible by p.

(a) Verify this for $p = 5$ and $a = 3$.

(b) Verify this for $p = 7$ and $a = 2$.

36. Every odd prime can be expressed as the difference of two squares in one and only one way.

(a) Find this one way for the prime number 5.

(b) Find this one way for the prime number 11.

37. There is only one solution in natural numbers for $a^2 + 2 = b^3$, and it is $a = 5$, $b = 3$. Verify this solution. $5^2 + 2 = 27 = 3^3$

38. There are only two solutions in integers for $a^2 + 4 = b^3$. One solution is $a = 2$, $b = 2$. Find the other solution.

The first four perfect numbers were identified in the text: 6, 28, 496, and 8128. The next two are 33,550,336 and 8,589,869,056. Use this information about perfect numbers to work Exercises 39–41.

39. Verify that each of these six perfect numbers ends in either 6 or 28. (In fact, this is true of all even perfect numbers.) Answers will vary.

40. Is conjecture (1) in the text (that the nth perfect number contains exactly n digits) true or false? Explain.

41. Is conjecture (2) in the text (that the even perfect numbers end in the digits 6 and 8, alternately) true or false? Explain.

According to the Web site www.shyamsundergupta.com/amicable.htm, a natural number is **happy** *if the process of repeatedly summing the squares of its decimal digits finally ends in 1. For example, the least natural number (greater than 1) that is happy is 7, as shown here.*

$$7^2 = 49, \quad 4^2 + 9^2 = 97, \quad 9^2 + 7^2 = 130,$$

$$1^2 + 3^2 + 0^2 = 10, \quad 1^2 + 0^2 = 1.$$

An amicable pair is a **happy amicable pair** *if and only if both members of the pair are happy numbers. (The first 5000 amicable pairs include only 111 that are happy amicable pairs.) For each amicable pair, determine whether neither, one, or both of the members are happy, and whether the pair is a happy amicable pair.*

42. 220 and 284

43. 1184 and 1210 one; not happy

44. 10,572,550 and 10,854,650

45. 35,361,326 and 40,117,714 both; happy

46. If the early Greeks knew the form of all even perfect numbers, namely $2^{n-1}(2^n - 1)$, then why did they not discover all the ones that are known today?

47. Explain why the primorial formula $p\# \pm 1$ does not result in a pair of twin primes for the prime value $p = 2$. Answers will vary.

48. **(a)** What two numbers does the primorial formula produce for $p = 7$?

(b) Which, if either, of these numbers is prime? 211 only

49. Choose the correct completion: The primorial formula produces twin primes

A. never. **B.** sometimes. **C.** always. B

See the margin note (on **page 195***) defining a Sophie Germain prime, and complete this table.*

	p	$2p+1$	Is p a Sophie Germain prime?
50.	2	5	___
51.	3	7	yes
52.	5	11	___
53.	7	15	no

Factorial primes *are of the form $n! \pm 1$ for natural numbers n. ($n!$ denotes "n factorial," the product of all natural numbers up to n, not just the primes as in the primorial primes. For example, $4! = 1 \cdot 2 \cdot 3 \cdot 4 = 24$.) As of late 2014, the largest verified factorial prime was $150{,}209! + 1$, which has 712,355 digits. Find the missing entries in the following table.*

	n	$n!$	$n! - 1$	$n! + 1$	Is $n! - 1$ prime?	Is $n! + 1$ prime?
	2	2	1	3	no	yes
54.	3	___	___	___	___	___
55.	4	24	23	25	yes	no
56.	5	___	___	___	___	___

57. Explain why the factorial prime formula does not give twin primes for $n = 2$. Answers will vary.

Based on the preceding table, complete each statement with one of the following:

A. *never,* **B.** *sometimes, or* **C.** *always.*

When applied to particular values of n, the factorial prime formula $n! \pm 1$ produces

58. no primes ___

59. exactly one prime B

60. twin primes ___

Because it does not equal the sum of its proper divisors, an abundant number is not perfect, but it is called ***pseudoperfect*** *if it is equal to the sum of a subset of its proper divisors. For example, 12, an abundant number, has proper divisors 1, 2, 3, 4, and 6. $12 \neq 1 + 2 + 3 + 4 + 6$, but $12 = 2 + 4 + 6$. (Also, $12 = 1 + 2 + 3 + 6$.)*

Show that each number is pseudoperfect.

61. 18 (Show it with two different sums.)
$18 = 1 + 2 + 6 + 9 = 3 + 6 + 9$

62. 24 (Show it with five different sums.)

Abundant numbers are so commonly pseudoperfect that when we find one that isn't, we call it **weird.** *There are no weird numbers less than 70. Do the following exercises to investigate 70.*

63. Show that 70 is abundant.
$1 + 2 + 5 + 7 + 10 + 14 + 35 = 74 > 70$

64. Show that 70 is *not* pseudoperfect and must therefore be the smallest weird number. (Among the first 10,000 counting numbers, there are only seven weird ones: 70, 863, 4030, 5830, 7192, 7912, and 9272.)

The following number is known as Belphegor's prime.

One nonillion, sixty-six quadrillion, six hundred trillion, one

Belphegor, who is referred to in many literary works, is one of the seven princes of hell, known for tempting men toward particular evils. Exercises 65–68 refer to Belphegor's prime.

65. Write out the decimal form of the number. (It is a "palindrome"—that is, it reads the same backward and forward. And in keeping with its name, it actually is a prime.) 1,000,000,000,000,066,600,000,000,000,001

66. What are the "middle three" digits? (The biblical book of Revelation calls this "the number of the beast.")

67. How many zeroes are on each side of the middle three digits? 13

68. With what digit does Belphegor's prime begin and end?

$$\begin{array}{r|rrrr} 2 & 2 & 3 & 5 & 6 \\ 3 & 1 & 3 & 5 & 3 \\ 5 & 1 & 1 & 5 & 1 \\ \hline & 1 & 1 & 1 & 1 \end{array} \quad \text{LCM} = 2 \cdot 3 \cdot 5 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 30$$

Using 30 as the least common denominator, we obtain

$$\frac{6}{5} + \frac{16}{3} + \frac{5}{6} + \frac{3}{2} = \frac{6}{5} \cdot \frac{6}{6} + \frac{16}{3} \cdot \frac{10}{10} + \frac{5}{6} \cdot \frac{5}{5} + \frac{3}{2} \cdot \frac{15}{15}$$

$$= \frac{36}{30} + \frac{160}{30} + \frac{25}{30} + \frac{45}{30}$$

$$= \frac{36 + 160 + 25 + 45}{30}$$

$$= \frac{266}{30} = 8\frac{26}{30}.$$

The combined total is $8\frac{26}{30}$ ounces, or, in decimal form, about 8.867 ounces.

5.4 EXERCISES

Decide whether each statement is true *or* false.

1. No two even natural numbers can be relatively prime. true

2. Two different prime numbers must be relatively prime.

3. If p is a prime number, then the greatest common factor of p and p^2 is p^2. false

4. If p is a prime number, then the least common multiple of p and p^2 is p^3.

5. There is no prime number p such that the greatest common factor of p and 2 is 2. false

6. The set of all common multiples of two given natural numbers is infinite.

7. Any two natural numbers have at least one common factor. true

8. The least common multiple of two different primes is their product.

9. No two composite numbers can be relatively prime. false

10. The product of any two natural numbers is equal to the product of their least common multiple and their greatest common factor.

Use the prime factors method to find the greatest common factor of each group of numbers.

11. 84 and 140 28

12. 315 and 90

13. 275 and 132 11

14. 264 and 504

15. 68, 102, and 425 17

16. 765, 780, and 990

Use the method of dividing by prime factors to find the greatest common factor of each group of numbers.

17. 150 and 260 10

18. 237 and 395

19. 600 and 90 30

20. 330 and 255

21. 84, 90, and 210 6

22. 585, 1680, and 990

Use the Euclidean algorithm to find the greatest common factor of each group of numbers.

23. 18 and 60 6

24. 77 and 84

25. 36 and 90 18

26. 72 and 90

27. 945 and 450 45

28. 200 and 350

Use the prime factors method to find the least common multiple of each group of numbers.

29. 48 and 60 240

30. 21 and 35

31. 81 and 45 405

32. 84 and 98

33. 20, 30, and 50 300

34. 15, 21, and 45

Use the method of dividing by prime factors to find the least common multiple of each group of numbers.

35. 27 and 36 108

36. 21 and 56

37. 63 and 99 693

38. 16, 120, and 216

39. 48, 54, and 60 2160

40. 154, 165, and 2310

Use the formula given in the text on ***page 203*** *and the results of* ***Exercises 23–28*** *to find the least common multiple of each group of numbers.*

41. 18 and 60 180

42. 77 and 84

43. 36 and 90 180

44. 72 and 90

45. 945 and 450 9450

46. 200 and 350

47. Explain in your own words how to find the greatest common factor of a group of numbers. Answers will vary.

48. Explain in your own words how to find the least common multiple of a group of numbers.

49. If p, q, and r are different primes, and a, b, and c are natural numbers such that $a < b < c$,

(a) what is the greatest common factor of $p^a q^c r^b$ and $p^b q^a r^c$? $p^a q^a r^b$

(b) what is the least common multiple of $p^b q^a$, $q^b r^c$, and $p^a r^b$? $p^b q^b r^c$

50. Find **(a)** the greatest common factor and **(b)** the least common multiple of $2^{25} \cdot 5^{17} \cdot 7^{21}$ and $2^{28} \cdot 5^{22} \cdot 7^{13}$. Leave your answers in prime factored form.

It is possible to extend the Euclidean algorithm in order to find the greatest common factor of more than two numbers. For example, if we wish to find the greatest common factor of 150, 210, and 240, we can first use the algorithm to find the greatest common factor of two of these (say, for example, 150 and 210). Then we find the greatest common factor of that result and the third number, 240. The final result is the greatest common factor of the original group of numbers.

Use the Euclidean algorithm as just described to find the greatest common factor of each group of numbers.

51. 90, 105, and 315 15

52. 48, 315, and 450

53. 144, 180, and 192 12

54. 180, 210, and 630

55. Suppose that the least common multiple of p and q is pq. What can we say about p and q?
p and q are relatively prime.

56. Suppose that the least common multiple of p and q is q. What can we say about p and q?

57. Recall some of your early experiences in mathematics (for example, in the elementary grade classroom). What topic involving fractions required the use of the least common multiple? Give an example. Answers will vary.

58. Recall some of your experiences in elementary algebra. What topics required the use of the greatest common factor? Give an example.

Refer to ***Examples 9 and 10*** *to solve each problem.*

59. *Inspecting Calculators* Jameel and Fahima work on an assembly line, inspecting electronic calculators. Jameel inspects the electronics of every sixteenth calculator, while Fahima inspects the workmanship of every thirty-sixth calculator. If they both start working at the same time, which calculator will be the first that they both inspect? 144th

60. *Night Off for Security Guards* Tomas and Jenny work as security guards at a factory. Tomas has every sixth night off, and Jenny has every tenth night off. If both are off on July 1, what is the next night that they will both be off together?

61. *Stacking Coins* Suyín has 240 pennies and 288 nickels. She wants to place them all in stacks so that each stack has the same number of coins, and each stack contains only one denomination of coin. What is the greatest number of coins that she can place in each stack?
48 (5 stacks of 48 pennies, 6 stacks of 48 nickels)

62. *Bicycle Racing* Aki and Felipe are in a bicycle race, following a circular track. If they start at the same place and travel in the same direction, and Aki completes a revolution every 40 seconds, which Felipe takes 45 seconds to complete each revolution, how long will it take them before they reach the starting point again simultaneously?

63. *Selling Books* Azad sold some books at \$24 each and used the money to buy some concert tickets at \$50 each. He had no money left over after buying the tickets. What is the least amount of money he could have earned from selling the books? What is the least number of books he could have sold? \$600; 25 books

64. *Sawing Lumber* Terri has some pieces of two-by-four lumber. Some are 60 inches long, and some are 72 inches long. All of them must be sawn into shorter pieces. If all sawn pieces must be the same length, what is the longest such piece so that no lumber is left over?