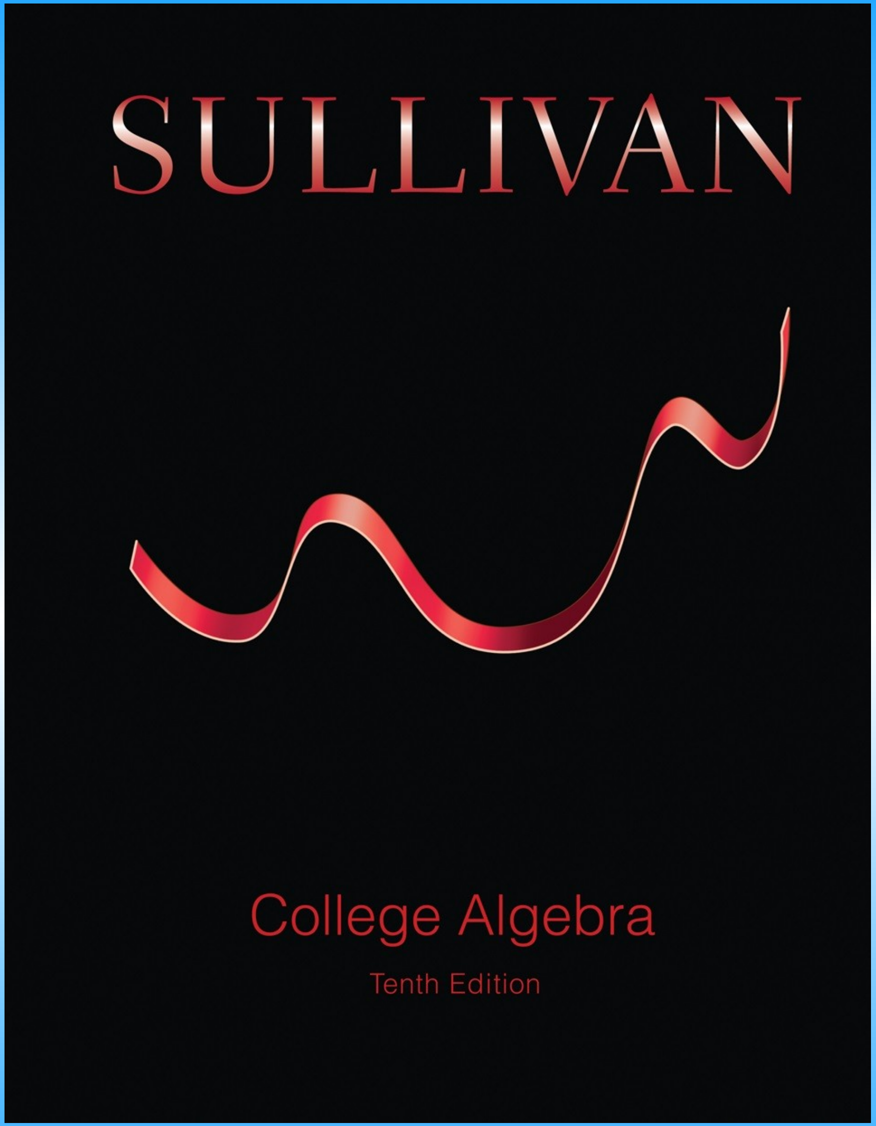
# Chapter 1

## Section 1



# 1.1 Linear Equations

**PREPARING FOR THIS SECTION** *Before getting started, review the following:*

- Properties of Real Numbers (Section R.1, pp. 9–13)
- Domain of a Variable (Section R.2, p. 21)

**Now Work** the **'Are You Prepared?'** problems on page 90.

**OBJECTIVES**

1. Solve a Linear Equation (p. 84)
2. Solve Equations That Lead to Linear Equations (p. 86)
3. Solve Problems That Can Be Modeled by Linear Equations (p. 87)

One method for solving an equation is to replace the original equation by a succession of **equivalent equations**, equations having the same solution set, until an equation with an obvious solution is obtained.

## Procedures That Result in Equivalent Equations

**1.** Interchange the two sides of the equation:

Replace $3 = x$ by $x = 3$

**2.** Simplify the sides of the equation by combining like terms, eliminating parentheses, and so on:

$$\text{Replace} \quad (x + 2) + 6 = 2x + (x + 1)$$
$$\text{by} \quad x + 8 = 3x + 1$$

**3.** Add or subtract the same expression on both sides of the equation:

$$\text{Replace} \quad 3x - 5 = 4$$
$$\text{by} \quad (3x - 5) + 5 = 4 + 5$$

**4.** Multiply or divide both sides of the equation by the same nonzero expression:

$$\text{Replace} \quad \frac{3x}{x-1} = \frac{6}{x-1} \qquad x \neq 1$$
$$\text{by} \quad \frac{3x}{x-1} \cdot (x - 1) = \frac{6}{x-1} \cdot (x - 1)$$

**5.** If one side of the equation is 0 and the other side can be factored, then we may use the Zero-Product Property* and set each factor equal to 0:

$$\text{Replace} \quad x(x - 3) = 0$$
$$\text{by} \quad x = 0 \quad \text{or} \quad x - 3 = 0$$

# Example

## Solving an Equation

Solve the equation: $3x - 5 = 4$

# Solution

Replace the original equation by a succession of equivalent equations.

$$3x - 5 = 4$$

$$(3x - 5) + 5 = 4 + 5 \quad \text{Add 5 to both sides.}$$

$$3x = 9 \quad \text{Simplify.}$$

$$\frac{3x}{3} = \frac{9}{3} \quad \text{Divide both sides by 3.}$$

$$x = 3 \quad \text{Simplify.}$$

The last equation, $x = 3$, has the single solution 3. All these equations are equivalent, so 3 is the only solution of the original equation, $3x - 5 = 4$.

# Solution continued

**Check:** Check the solution by substituting 3 for $x$ in the original equation.

$$\begin{aligned} 3x - 5 &= 4 \\ 3(3) - 5 &\stackrel{?}{=} 4 \\ 9 - 5 &\stackrel{?}{=} 4 \\ 4 &= 4 \end{aligned}$$

The solution checks. The solution set is $\{3\}$.

## Steps for Solving Equations

**STEP 1:** List any restrictions on the domain of the variable.

**STEP 2:** Simplify the equation by replacing the original equation by a succession of equivalent equations using the procedures listed earlier.

**STEP 3:** If the result of Step 2 is a product of factors equal to 0, use the Zero-Product Property and set each factor equal to 0 (procedure 5).

**STEP 4:** Check your solution(s).

# Solve a Linear Equation

# Definition

A **linear equation in one variable** is an equation equivalent in form to

$$ax + b = 0$$

where $a$ and $b$ are real numbers and $a \neq 0$.

# Example

**Solving a Linear Equation**

Solve the equation: $\frac{1}{2}(x+5)-4=\frac{1}{3}(2x-1)$

# Solution

To clear the equation of fractions, multiply both sides by 6, the least common multiple (LCM) of the denominators of the fractions $\frac{1}{2}$ and $\frac{1}{3}$.

$$\frac{1}{2}(x+5)-4=\frac{1}{3}(2x-1)$$

$$6\left[\frac{1}{2}(x+5)-4\right]=6\left[\frac{1}{3}(2x-1)\right]$$ Multiply both sides by 6, the LCM of 2 and 3.

$$3(x+5)-6\cdot 4=2(2x-1)$$ Use the Distributive Property on the left and the Associative Property on the right.

$$3x+15-24=4x-2$$ Use the Distributive Property.

$$3x-9=4x-2$$ Combine like terms.

$$3x-9+9=4x-2+9$$ Add 9 to each side.

$$3x=4x+7$$ Simplify.

$$3x-4x=4x+7-4x$$ Subtract $4x$ from each side.

$$-x=7$$ Simplify.

$$x=-7$$ Multiply both sides by $-1$.

# Solution continued

**Check:** Substitute $-7$ for $x$ in the expressions on the left and right sides of the original equation, and simplify. If the two expressions are equal, the solution checks.

$$\frac{1}{2}(x+5)-4=\frac{1}{2}(-7+5)-4=\frac{1}{2}(-2)-4=-1-4=-5$$

$$\frac{1}{3}(2x-1)=\frac{1}{3}[2(-7)-1]=\frac{1}{3}(-14-1)=\frac{1}{3}(-15)=-5$$

Since the two expressions are equal, the solution checks. The solution set is $\{-7\}$.

# Solve Equations That Lead to Linear Equations

# Example

**Solving an Equation That Leads to a Linear Equation**

Solve the equation: $(2y + 1)(y - 1) = (y + 5)(2y - 5)$

# Solution

$$(2y+1)(y-1) = (y+5)(2y-5)$$

| | |
|---|---|
| $2y^2 - y - 1 = 2y^2 + 5y - 25$ | Multiply and combine like terms. |
| $-y - 1 = 5y - 25$ | Subtract $2y^2$ from each side. |
| $-y = 5y - 24$ | Add 1 to each side. |
| $-6y = -24$ | Subtract 5y from each side. |
| $y = 4$ | Divide both sides by −6. |

✓**Check:** $(2y+1)(y-1) = [2(4)+1](4-1) = (8+1)(3) = (9)(3) = 27$

$(y+5)(2y-5) = (4+5)[2(4)-5] = (9)(8-5) = (9)(3) = 27$

The two expressions are equal, so the solution checks. The solution set is $\{4\}$.

# Example

**An Equation with No Solution**

Solve the equation: $\dfrac{3x}{x-1} + 2 = \dfrac{3}{x-1}$

# Solution

First, note that the domain of the variable is $\{x | x \neq 1\}$. Since the two quotients in the equation have the same denominator, $x - 1$, simplify by multiplying both sides by $x - 1$. The resulting equation is equivalent to the original equation, since we are multiplying by $x - 1$, which is not 0. (Remember, $x \neq 1$.)

$$\frac{3x}{x-1} + 2 = \frac{3}{x-1}$$

$$\left(\frac{3x}{x-1} + 2\right) \cdot (x-1) = \frac{3}{\cancel{x-1}} \cdot \cancel{(x-1)}$$ Multiply both sides by $x - 1$; cancel on the right.

$$\frac{3x}{\cancel{x-1}} \cdot \cancel{(x-1)} + 2 \cdot (x-1) = 3$$ Use the Distributive Property on the left side; cancel on the left.

$$3x + 2x - 2 = 3$$ Simplify.

$$5x - 2 = 3$$ Combine like terms.

$$5x = 5$$ Add 2 to each side.

$$x = 1$$ Divide both sides by 5.

The solution appears to be 1. But recall that $x = 1$ is not in the domain of the variable, so this value must be discarded. The equation has no solution. The solution set is $\varnothing$.

# Solve Problems That Can Be Modeled by Linear Equations

## Steps for Solving Applied Problems

**STEP 1:** Read the problem carefully, perhaps two or three times. Pay particular attention to the question being asked in order to identify what you are looking for. Identify any relevant formulas you may need ($d = rt$, $A = \pi r^2$, etc.). If you can, determine realistic possibilities for the answer.

**STEP 2:** Assign a letter (variable) to represent what you are looking for, and, if necessary, express any remaining unknown quantities in terms of this variable.

**STEP 3:** Make a list of all the known facts, and translate them into mathematical expressions. These may take the form of an equation (or, later, an inequality) involving the variable. The equation (or inequality) is called the **model**. If possible, draw an appropriately labeled diagram to assist you. Sometimes a table or chart helps.

**STEP 4:** Solve the equation for the variable, and then answer the question, usually using a complete sentence.

**STEP 5:** Check the answer with the facts in the problem. If it agrees, congratulations! If it does not agree, try again.

# Example

**Investments**

A total of $18,000 is invested, some in stocks and some in bonds. If the amount invested in bonds is half that invested in stocks, how much is invested in each category?

# Solution

| | |
|---|---|
| **Step 1:** *Determine what you are looking for.* | We are being asked to find the amount of two investments. These amounts must total \$18,000. (Do you see why?) |
| **Step 2:** *Assign a variable to represent what you are looking for. If necessary, express any remaining unknown quantities in terms of this variable.* | If $x$ equals the amount invested in stocks, then the rest of the money, $18{,}000 - x$, is the amount invested in bonds. |
| **Step 3:** *Translate the English into mathematical statements. It may be helpful to draw a figure that represents the situation. Sometimes a table can be used to organize the information. Use the information to build your model.* | Set up a table: (see table below) We also know that: Total amount invested in bonds is one-half that in stocks |

Set up a table:

| Amount in Stocks | Amount in Bonds | Reason |
|---|---|---|
| $x$ | $18{,}000 - x$ | Total invested is \$18,000. |

We also know that:

Total amount invested in bonds is one-half that in stocks

$$18{,}000 - x = \frac{1}{2}(x)$$

# Solution continued

**Step 4:** Solve the equation and answer the original question.

$$18{,}000 - x = \frac{1}{2}x$$

$$18{,}000 = x + \frac{1}{2}x \qquad \text{Add } x \text{ to both sides.}$$

$$18{,}000 = \frac{3}{2}x \qquad \text{Simplify.}$$

$$\left(\frac{2}{3}\right)18{,}000 = \left(\frac{2}{3}\right)\left(\frac{3}{2}x\right) \qquad \text{Multiply both sides by } \frac{2}{3}.$$

$$12{,}000 = x \qquad \text{Simplify.}$$

So \$12,000 is invested in stocks, and \$18,000 − \$12,000 = \$6000 is invested in bonds.

**Step 5:** Check your answer with the facts presented in the problem.

The total invested is \$12,000 + \$6000 = \$18,000, and the amount in bonds, \$6000, is half that in stocks, \$12,000.