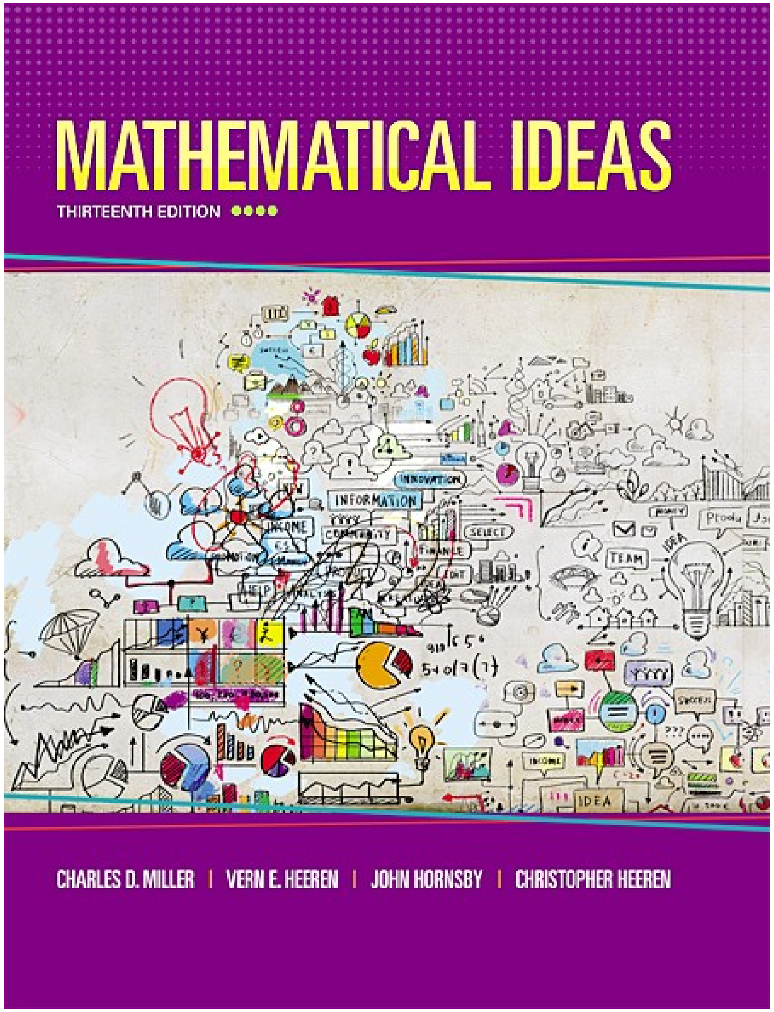
Chapter 7

The Basic Concepts of Algebra



Chapter 7: The Basic Concepts of Algebra

Section 7-1

Linear Equations

Linear Equations

- Solve a linear equation.
- Identify a linear equation as a conditional equation, an identity, or a contradiction.
- Solve a literal equation or a formula for a specified variable.
- Use a linear model in an application.

Algebraic Expression

An **algebraic expression** involves only the basic operations of addition, subtraction, multiplication, or division (except by 0), or raising to powers or taking roots on any collection of variables and numbers.

$$4x - 7, \quad \sqrt{m} + 3, \quad \text{and} \quad \frac{xy^3z}{k}$$

Algebraic expressions

Equation

An **equation** is a statement that two algebraic expressions are equal. A *linear equation in one variable* involves only real numbers and one variable.

$$4x - 7 = 8 \quad \text{and} \quad 5k + 3 = k + 1$$

Linear equations

Linear Equation in One Variable

An equation in the variable x is a **linear equation** if it can be written in the form

$$\boldsymbol{Ax + B = C}$$

where A, B, and C are real numbers, with $A \neq 0$.

Terminology

A linear equation in one variable is also called a **first-degree equation**.

If the variable in an equation is replaced by a real number that makes the statement of the equation true, then that number is a **solution** of the equation. An equation is **solved** by finding its **solution set**, the set of all answers.

Equivalent equations are equations with the same solution set.

Addition Property of Equality

For all real numbers A, B, and C, the equations

$$A = B \quad \text{and} \quad A + C = B + C$$

are equivalent. (The same number may be added to both sides of an equation without changing the solution set.)

Multiplication Property of Equality

For all real numbers A, B, and C, where $C \neq 0$, the equations

$$A = B \quad \text{and} \quad AC = BC$$

are equivalent. (Both sides of an equation may be multiplied by the same nonzero number without changing the solution set.)

Solving a Linear Equation in One Variable

Step 1 **Clear fractions.** Eliminate any fractions by multiplying both sides of the equation by a common denominator.

Step 2 **Simplify each side separately**. Use the distributive property to clear parentheses, and combine like terms as needed.

Step 3 **Isolate the variable terms on one side**. Use the addition property of equality.

Solving a Linear Equation in One Variable

Step 4 **Transform so that the coefficient of the variable is 1**. Use the multiplication property of equality.

Step 5 **Check**. Substitute the solution into the original equation.

Example: Using the Distributive Property to Solve a Linear Equation

Solve $2(x - 5) + 3x = x + 6$.

Solution

$2x - 10 + 3x = x + 6$	Distributive property.
$5x - 10 = x + 6$	Combine like terms.
$5x - 10 + 10 = x + 6 + 10$	Add 10.
$5x = x + 16$	Combine like terms.

Example: Using the Distributive Property to Solve a Linear Equation

Solution (continued)

$$5x = x + 16$$

$$5x - x = x + 16 - x \quad \text{Subtract } x.$$

$$4x = 16 \quad \text{Combine like terms.}$$

$$\frac{4x}{4} = \frac{16}{4} \quad \text{Divide by 4.}$$

$$x = 4 \quad \text{Simplify.}$$

Check that the solution set is {4} by substituting 4 for x in the original equation.

Example: Solving a Linear Equation with Fractions

Solve $\dfrac{x+7}{6}+\dfrac{2x-8}{2}=-4.$

Solution

$$6\left(\frac{x+7}{6}+\frac{2x-8}{2}\right)=6(-4)$$ Multiply by LCD, 6.

$$6\left(\frac{x+7}{6}\right)+6\left(\frac{2x-8}{2}\right)=6(-4)$$ Distributive property.

$$x+7+3(2x-8)=-24$$ Multiply.

Example: Solving a Linear Equation with Fractions

Solution (continued)

$x + 7 + 6x - 24 = -24$	Distributive property.
$7x - 17 = -24$	Combine like terms.
$7x - 17 + 17 = -24 + 17$	Add 17.
$7x = -7$	Combine like terms.
$\frac{7x}{7} = \frac{-7}{7}$	Divide by 7.
$x = -1$	

Now check.

Types of Linear Equations

Type	Number of Solutions	Final Line When Solving
Conditional	One	x = number
Identity	Infinite; solution set {all real numbers}	True statement, such as 0 = 0.
Contradiction	None; solution set $\varnothing$	False statement, such as 0 = 1.

Example: Contradiction

Solve $3k + 4 - 2k = k$.

Solution

$k + 4 = k$	Combine like terms.
$k - k + 4 = k - k$	Subtract k.
$4 = 0$	Combine like terms.

Because the result is *false*, the equation has no solution. The solution set is $\emptyset$, so the original equation is a contradiction.

Example: Identity

Solve $3k + 4 - 2k = k + 4$.

Solution

$k + 4 = k + 4$	Combine like terms.
$k - k + 4 = k - k$	Subtract *k*.
$4 = 4$	Combine like terms.

Because the result is *true*, the solution set is {all real numbers} and the original equation is an identity.

Literal Equations and Formulas

An equation involving *variables* (or letters), such as $y = mx + b$ is called a **literal equation**.

Formulas are literal equations in which more than one letter is used to express a relationship.

It may be necessary to solve for one of the variables in a formula. This process is called **solving for a specified variable**.

Solving for a Specified Variable

Step 1 If the equation contains fractions, multiply both sides by the LCD to clear the fractions.

Step 2 Transform so that all terms with the specified variable are on one side and all terms without that variable are on the other side.

Step 3 Divide each side by the factor that is the coefficient of the specified variable.

Example: Solving for a Specified Variable

Solve the formula $P = 2L + 2W$ for L.

Solution

$$P - 2W = 2L + 2W - 2W$$ Subtract $2W$.

$$P - 2W = 2L$$ Combine like terms.

$$\frac{P - 2W}{2} = \frac{2L}{2}$$ Divide by 2.

$$\frac{P - 2W}{2} = L \quad \text{or} \quad \frac{P}{2} - W = L.$$

Models

A **mathematical model** is an equation (or inequality) that describes the relationship between two quantities. A *linear model* is a linear equation.

Example: Temperature Conversion

The relationship between degrees Celsius (C) and degrees Fahrenheit (F) is modeled by the linear equation

$$F = \frac{9}{5}C + 32$$

Find the Fahrenheit temperature that corresponds to 30 degrees Celsius.

Example: Temperature Conversion

Solution

Because $C = 30$, the equation becomes

$$F = \frac{9}{5}C + 32$$

$$F = \frac{9}{5}(30) + 32$$

$$F = 54 + 32$$

$$F = 86$$

The Celsius temperature 30 degrees corresponds to 86 degrees Fahrenheit.