

AS1 (Assignment 1, Unit 3): Z-scores and Probability

Please type your answers in red 😊

1. What information is provided by the sign (+/-) of a z-score? A positive z-score means the score is above the mean and a negative z-score means it falls below the mean.
2. What information is provided by the numerical value of the z-score? The number value of the z-score tells you the distance from the mean in units of standard deviation.
3. A distribution has a standard deviation of $\sigma = 10$. Find the z-score for each of the following locations in the distribution.
 - a. Above the mean by 5 points.
Answer: 5
 - b. Above the mean by 2 points.
Answer: 2
 - c. Below the mean by 20 points.
Answer: -20
 - d. Below the mean by 15 points.
Answer: 15
4. For a distribution with a standard deviation of $\sigma = 20$, describe the location of each of the following z-scores in terms of its position relative to the mean. For example, $+1.00$ is a location that is 20 points above the mean.
 - a. $z = +2.00$
Answer: 40
 - b. $z = +.50$
Answer: 10
 - c. $z = -1.00$
Answer: -20
 - d. $z = -0.25$
Answer:

5. For a population with $\mu = 80$ and $\sigma = 10$,

- a. Find the z -score for each of the following X values. (Note: You should be able to find these values using the definition of a z -score. You should not need to use a formula or do any serious calculations.)

$$X = 75: z = \underline{-0.5} \quad X = 100: z = \underline{2} \quad X = 60: z = \underline{-2}$$

$$X = 95: z = \underline{1.5} \quad X = 50: z = \underline{-3} \quad X = 85: z = \underline{0.5}$$

- b. Find the score (X value) that corresponds to each of the following z -scores. (Again, you should not need a formula or any serious calculations.)

$$z = 1.00: X = \underline{90} \quad z = 0.20: X = \underline{82} \quad z = 1.50: X = \underline{95}$$

$$z = -0.50: X = \underline{75} \quad z = -2.00: X = \underline{60} \quad z = -1.50: X = \underline{65}$$

6. For a population with a mean of $\mu = 40$ and $\sigma = 11$, find the z -score for each of the following X values. (Note: You probably will need to use a formula and a calculator to find these values)

$$X = 45: z = \underline{0.45} \quad X = 52: z = \underline{1.09} \quad X = 41: z = \underline{0.09}$$

$$X = 30: z = \underline{-0.9} \quad X = 25: z = \underline{-1.36} \quad X = 38: z = \underline{-0.18}$$

7. For a population with a mean of $\mu = 100$ and a standard deviation of $\sigma = 20$,

- a. Find the z -score for each of the following X values.

$$X = 108: z = \underline{0.4} \quad X = 115: z = \underline{0.75} \quad X = 130: z = \underline{1.5}$$

$$X = 90: z = \underline{-0.5} \quad X = 88: z = \underline{-0.6} \quad X = 95: z = \underline{-0.25}$$

- b. Find the score (X value) that corresponds to each of the following z -scores.

$$z = -0.40: X = \underline{92} \quad z = -0.50: X = \underline{90} \quad z = 1.80: X = \underline{136}$$

$$z = 0.75: X = \underline{115} \quad z = 1.50: X = \underline{130} \quad z = -1.25: X = \underline{75}$$

8. A population has a mean of $\mu = 60$ and a standard deviation of $\sigma = 12$.
- a. For the population, find the z-score for each of the following X values.

$$X = 69: z = \underline{0.75} \quad X = 84: z = \underline{2} \quad X = 63: z = \underline{0.25}$$

$$X = 54: z = \underline{-0.5} \quad X = 48: z = \underline{-1} \quad X = 45: z = \underline{-1.25}$$

- b. For the same population, find the score (X value) that corresponds to each of the following z-scores.

$$z = 0.50: X = \underline{66} \quad z = 1.50: X = \underline{78} \quad z = -2.50: X = \underline{30}$$

$$z = -0.25: X = \underline{57} \quad z = -0.50: X = \underline{54} \quad z = 1.25: X = \underline{75}$$

9. A sample has a mean of $M = 30$ and a standard deviation of $s = 8$. Find the z-score of for each of the following X values from this sample.

$$X = 32: z = \underline{0.25} \quad X = 34: z = \underline{0.5} \quad X = 36: z = \underline{0.75}$$

$$X = 28: z = \underline{-0.25} \quad X = 20: z = \underline{-1.25} \quad X = 18: z = \underline{-1.5}$$

10. A sample has a mean of $M = 25$ and a standard deviation of $s = 5$. For this sample, find the X value corresponding to each of the following z-scores.

$$z = 0.40: X = \underline{27} \quad z = 1.20: X = \underline{31} \quad z = 2.00: X = \underline{35}$$

$$z = -0.80: X = \underline{21} \quad z = -0.60: X = \underline{22} \quad z = -1.40: X = \underline{18}$$

11. Find the z-score corresponding to a score of $X = 45$ of the following distributions.

a. $\mu = 40$ and $\sigma = 20$

Answer: 0.25

b. $\mu = 40$ and $\sigma = 10$

Answer: 0.5

c. $\mu = 40$ and $\sigma = 5$

Answer: 1

d. $\mu = 40$ and $\sigma = 2$

Answer: 2.5

Find the X value corresponding to $z = 0.25$ for each of the following distributions.

e. $\mu = 40$ and $\sigma = 4$Answer: 41

f. $\mu = 40$ and $\sigma = 8$Answer: 42

g. $\mu = 40$ and $\sigma = 16$Answer: 44

h. $\mu = 40$ and $\sigma = 32$Answer: 48

12. For each of the following populations, would a score of $X = 50$ be considered a central score (near the middle of the distribution) or an extreme score (far out in the tail of the distribution)?

a. $\mu = 45$ and $\sigma = 10$Answer: _____

b. $\mu = 45$ and $\sigma = 2$Answer: _____

c. $\mu = 90$ and $\sigma = 20$Answer: _____

d. $\mu = 60$ and $\sigma = 20$Answer: _____

13. A distribution of exam scores has a mean of $\mu = 78$.

a. If your score is $X = 70$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: 8

b. If your score is $X = 80$, which standard deviation would give you a better grade: $\sigma = 4$ or $\sigma = 8$?

Answer: 4

14. For each of the following, identify the exam score that should lead to the better grade.

a. A score of $X = 74$ on an exam with $M = 82$ and $\sigma = 8$; or a score of $X = 40$ on an exam with $\mu = 50$ and $\sigma = 20$.

Answer: 40

b. A score of $X = 51$ on an exam with $\mu = 45$ and $\sigma = 2$; or a score of $X = 90$ on an exam with $\mu = 70$ and $\sigma = 20$.

Answer: 51

c. A score of $X = 62$ on an exam with $\mu = 50$ and $\sigma = 8$; or a score of $X = 23$ on an exam with $\mu = 20$ and $\sigma = 2$ Answer: Both

15. A population consists of the following $N = 5$ scores: 0, 6, 4, 3, and 12.

a. Compute μ and σ for the population

$$\text{Mean} = \underline{5} \quad \text{Standard deviation} = \underline{8.9}$$

b. Find the z-score for each score in the population

$$X=0, z = \underline{-0.56} \quad X=6, z = \underline{0.11} \quad X=4, z = \underline{-0.11} \quad X=3, z = \underline{-0.22} \quad X=12, z = \underline{0.07865}$$

15. A sample consists of the following $n = 7$ scores: 5, 0, 4, 5, 1, 2, and 4.

a. Compute the mean and standard deviation for the sample.

$$\text{Mean} = \underline{3} \quad \text{Standard deviation} = \underline{4.8}$$

b. Find the z-score for each score in the sample.

$$X=5, z = \underline{-0.4} \quad X=0, z = \underline{-0.625} \quad X=4, z = \underline{0.2} \quad X=5, z = \underline{0.41} \quad X=1, z = \underline{0.4}$$

$$X=2, z = \underline{-0.2} \quad X=4, z = \underline{0.2}$$

16. There are 20 black marbles, 40 blue marbles, and 5 red marbles in a jar.

a. What is the probability of selecting one red marble? _____

b. What is the probability of selecting one black marble? _____

c. What is the probability of selecting one blue marble? _____

d. Which has the highest probability of being selected? _____

e. Which has the lowest probability of being selected? _____

17. Probability values range from _____ to _____.

18. Please describe at least three characteristics of the NORMAL DISTRIBUTION

1. It is symmetrical in shape

2. All measure of central tendency are located at the center / have the same value

3. There are more score located near the mean than there are away from the mean

19. In a normal population of IQ scores, what percent of people have “average” IQ’s?

Answer 68.13%

20. In a normal distribution, what percentage of people would be located at or below 2 standard deviations from the mean?

Answer 2.28%

21. Answer the following questions based on a distribution with a $\mu = 30$ and $\sigma = 5$:

- a. What range of scores is considered “average”? _____ to _____
- b. What percentage of people has an average score? _____
- c. What percentage of people has extremely high or extremely low scores? _____
- d. What *range of scores* (requires numbers to be noted in the blank spaces) have the highest probability of being selected? _____ to _____