BUS 333: Finance I
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NOTES ON THE MATHEMATICS OF FINANCE

4. Present Value and the Discount Factor

In Section 3 we defined the discount factor *v* so that when we multiply a payment *M* that is scheduled to occur in one year by the discount factor, we get the present value (aka discounted value) of that payment. That is, the present value equals *vM.*

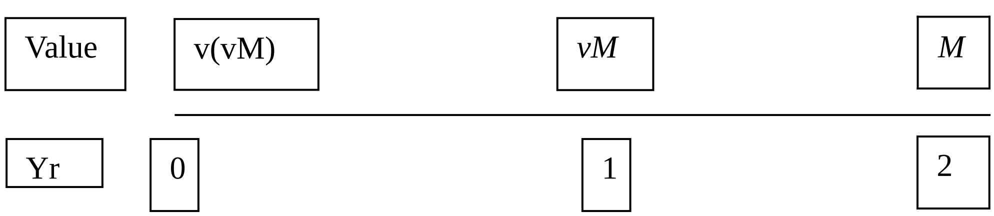
In this section we will use the discount factor to compute the present value of payments that occur at other points in the future, not just in one year. For example, what is the value today of a $500 Savings bond that matures in five years?

Also, in this section we will show how to modify the discount factor to handle those situations where interest is compounded more frequently than annually. We will see that we lose nothing in the ease of working with the discount factor to find present value in such cases. Before we get to this, let's consider how to compute the present value of payments at any future time.

Present Value at any Future Time t

From Section 3 we know that the present value of a payment of *M* one year from now is *vM.* But what is the present value if the payment is scheduled to occur in 2 years? We simply discount the payment one year to the find the discounted value one year before the payment occurs, and then discount it again to find the discounted value two years before the payment occurs. Figure 2 shows the value at the beginning of each year.

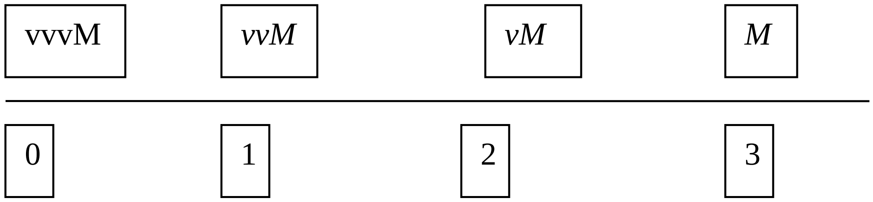
Figure 2.



So the present value is *v(vM),* normally written as v^2M .

What is the value if *M* is payable in 3 years?

Figure 3.



Again, *vvvM* is usually written as v^3M .

It shouldn't surprise you that the present value of *M* payable in 5 years is v^t .

In general, the present value of *M*, payable in *t* years is v^tM . The discount factor makes it a simple matter to compute the present value of a payment at any point in the future. For example, suppose you set a financial goal to retire at age 70 with a savings nest egg of $1,000,000. What is the present value of your savings goal? (Assume that you are age 20) $PV_{50} = v^{50} \times \$1,000,000$. When computed at 4% interest rate, *PV*=$140,713.

Mathematically, we express present value as follows:

$$PV_t = v^t M = \left(\frac{1}{1+i} \right)^t M$$

Note that present value depends on both time to payment and the interest rate available. The discount factor *v* is less than 1, so the present value is always less than 1. If interest rates increase, then the discount factor gets smaller and so the present value gets smaller too. Note also that since v is less than 1, then as time to payment increases, v^t gets smaller. These are basic relationships between time and value and interest rates and value that should become intuitively clear to you.

<u>Interest Compounded More Frequently than Annually</u>

Just as before in Section 2 of these Notes, we adjust our thinking from an annual basis to the same basis as the frequency at which interest is compounded. If interest is compounded quarterly, we think in terms of the interest rate <u>per quarter</u>, and we measure time in the number of quarters until the scheduled payment. If interest is compounded monthly, then we want the interest rate <u>per month</u>, and we measure the number of months to the scheduled payment. Here are some examples.

Suppose we want to have $1000 in the bank in three months. The bank pays interest at 8%, compounded quarterly. How much should we deposit today? (Answer: The interest rate per quarter is 2%. There is one quarter until the scheduled payment. So the value needed today is $v^1 \times \$1000 = \left(\frac{1}{1.02} \right)^1 \times \$1000 = 980.39$.)

Suppose we need that $1000 in one year. Same bank, same interest rate. What is the required deposit now? (Answer: $v^4 \times \$1000 = \left(\frac{1}{1.02}\right)^4 \times \$1000 = 923.85$)

Finally, suppose we want that $1000 in 6 ½ years. Same bank, same interest rate. (Answer: $v^{26} \times \$1000 = \left(\frac{1}{1.02}\right)^{26} \times \$1000 = \$597.58$)

Once again, the fact that interest is compounded more frequently than annual is not a big deal. But you must keep clearly in mind what you are doing, and do the computations with care. Here are some practice problems.

<u>Practice Problems</u>

1. What is the present value of a lease payment of $560,000, due in 6 months. Assume interest rates are 6%, compounded monthly.

   (Answer: $v^6 \times \$560,000 = \left(\frac{1}{1.005}\right)^6 \times \$560,000 = \$543,490.19$)
2. Suppose a company has to pay a contractor $200,000 one year from now, and $300,000 two years from now as part of a contract. If interest rates are 8.5%, what is the present value of that liability?

   (Answer: $v^1 \times 200,000 + v^2 \times 300,000 = \left(\frac{1}{1.085}\right)^1 \times 200,000 + \left(\frac{1}{1.085}\right)^2 \times 300,000$)

3. Suppose you must make a payment of $2,500 in five years. The bank is paying 6% promotional interest in the first year of deposit, and 5% interest after the first year. What is the amount of the deposit you should make today to have the required funds in 5 years?
   (Answer: In this problem we must discount the future payment four years at 5% and one year at 6%.

   $$PV = v_{6\%}^1 \times v_{5\%}^4 \$2,500 = \left(\frac{1}{1.06}\right)^1 \times \left(\frac{1}{1.05}\right)^4 \times \$2,500 = \$1,940.34$$