

NOTES ON THE MATHEMATICS OF FINANCE

3. The Discount Factor v

In this Section of Notes, we introduce a notation and methodology which will be most helpful to us when we want to solve the problem of finding the present value of a payment that will occur at some point in the future. For example, suppose you own a US savings bond that will mature for \$ 500 in one year. What is it worth today? While this may be a simple, almost trivial example, the technique that we will learn to solve this problem will help us to solve much more complex problems involving the present value of payments scheduled for the future.

The present value of our \$500 savings bond is less than \$500. The maturity value must be “discounted” to reflect that we won’t receive the maturity value for another year. The amount of the discount depends on the level of available interest rates. Suppose money can be invested for a year and earn 15% interest. Then the amount of the discount will reflect that waiting a year to receive the cash means that you cannot earn 15% interest in the next year. You will discount the value of the bond to reflect this lost opportunity. Imagine someone offered to buy the bond from you. **The least amount of money you would be willing to accept is the amount which, if you deposited it in a bank at 15% interest would give you \$500 at the end of the year.** Since 15% is a high rate, you would be willing to grant a big discount on the value of the bond, to get the cash now to invest at 15%.

However, if available interest rates are only 1% (as the economic situation is today), then you would not be willing to give a big discount on the value of the Savings bond. Why not? Because interest rates are so low, money you receive now and invest will only increase 1% over the next year. So you would only discount the value of the \$500 Savings bond a small amount.

That is the basic concept of discounting. Now let’s learn the details of the method developed to solve these problems, using something called the discount factor.

Defining the discount factor

Suppose a bank is offering to pay 4% interest on deposits. Then \$1.00 deposited today will be worth \$1.04 in one year’s time. That is to say that \$1.00 today is equivalent in value to \$1.04 one year from now. In still other words, we say that \$1.00 is the discounted value (or present value) of \$1.04 payable in one year.

Let’s define a discount factor called v . We want to use v in a simple way. **We want to define v in such a way that when we multiply the value of a payment in one year by v , we get the discounted value or present value of that future payment.**

(That's why it's called the discount factor.) In our example of the bank and 4% interest, we want it to be the case that:

$$1.00 = 1.04 \times v$$

Note that the left hand side is the value of our deposit today, and the right hand side is the value of the deposit a year from now, times the discount factor. For the two sides to be equivalent, it must be that:

$$v = \frac{1.00}{1.04}$$

We can now use v to find the discounted value of other payments that will occur in one year. For example, a car payment of \$624 due in one year would have a discounted value of \$600 (624 times $v = 600$). The \$1,000,000 maturity value of a corporate bond that matures in one year has a discounted value of \$961,538.46. (As you surmised, $v = .961538$) In general a payment of X , due in one year, has a discounted value (aka present value) of X times v .

Here are a couple more examples.

Payment due in one year:	Discount Factor (at 4%)	Discounted Value
\$ 1000	.961538	\$ 961.54
\$ 1.00	.961538	\$.96
\$ 12,456	.961538	\$ 11,976.92

(Don't move ahead until you are sure you understand this concept. The technique of a discount factor is used everywhere in finance. So let's start with a solid foundation.)

What if available interest rates are 8%, not 4%? In that case, \$1 today is equivalent to \$1.08 one year from now. The discount factor which makes the two payments

equivalent is therefore: $1.00 = 1.08 \times v$. In this case, $v = \frac{1.00}{1.08}$. When interest rates

are 8%, then \$624 due in one year has a discounted value (or present value) equal to \$577.74. (624 times v , where v is based on 8%, and has a value of $v = \frac{1.00}{1.08}$) Note

that the present value of \$624 is lower than when we calculated it before under an assumption of 4% interest. Earlier we got a present value of \$600, and now we get a present value of \$577.74. As I discussed in the intro to this section of notes, when rates are low (4%), there is less interest lost by waiting to receive the future payment, so the amount of the discount is small. When rates are higher (8%), then there is a larger amount of interest that could be earned, so the discount is greater, and the present value is smaller.

Mathematical Definition

Mathematically, the discount factor is defined by the equivalence relationship:

$$1 = (1 + i) \times v$$

The left hand side of the equation is the value of \$1 today (the present value). The right hand side of the equation is the value one year from now, times the discount factor v . For the two sides to be equivalent, v must be defined as follows:

$$v = \frac{1}{1 + i}$$

This is the mathematical definition of v . Note that v changes value for different interest rates. We understand why. The amount of the discount depends on the level of available interest rates, so the discount factor must also depend on the interest rate. When interest rates increase, the denominator in v gets larger, and therefore v gets smaller. When interest rates are smaller, then the denominator gets smaller and the discount factor gets larger. For interest rates very close to 0%, the discount factor is virtually 1, which means that there is almost no difference between future value and present value.

It will be very helpful to you if you thoroughly understand the concept of the discount factor, and how it changes with interest rates. As financial managers, you will often be dealing with issues related to changing interest rates, and a solid intuitive understanding will be very important to you.

Practice problems

1. A bank is offering 10% interest on deposits. What is the discount factor? How much do you need to deposit today to have \$750 in one year?
(Answer: \$ 681.82 : \$750 times v , where $v = .909091$)
2. Again, available interest rates are 10%. What is least amount you are willing to accept as payment today, instead of waiting one year to receive \$10,000?
(Answer: \$9,090.91)
3. Suppose interest rates are only 2%. Now how much are you willing to accept today, instead of waiting one year to receive \$10,000? What is the discount factor?
(Answer: \$9,803.92 or more; $v = .980392$)
4. Someone owes you \$1000, payable in one year. But you need cash now, so you offer to accept \$950, if payment is made now. What is the discount factor? What is the implied interest rate?
(Answer: The discount factor is .95. Using the definition of v , we can solve for the implied interest rate, which is 5.26%)

This section introduced the discount factor v . It is used to “discount” future payments to their present value. In the next section of notes, we will use v to solve many common finance problems.

