BUS 333: Finance I
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NOTES ON THE MATHEMATICS OF FINANCE

1. Compound Interest

Interest is the amount paid by a borrower to the lender on a specific amount of money over a specific period of time. Interest is usually expressed as a percentage rate, which is applied to the amount borrowed. For example, banks pay interest on deposits (money they borrow from you), and charge interest on loans they give to others (at a much higher interest rate). To fully define interest, you need to specify the amount on which interest is paid (the principal), the rate at which interest is charged (or earned, depending on which side of the transaction you sit), and the time period for which the interest rate is applicable (e.g., annual, monthly, etc.). Each period, interest is "compounded", meaning that the interest earned last period is added to the principal and earns interest on itself in the next period.

Basic Example: Suppose a bank pays 3% interest on deposits. Here is the progression of value on a $1000 deposit over a period of 5 years.

Year	Beg. Balance	Rate	Interest		Ending Balance	
1	$ 1,000.00	3.0%	$ 30.00		$ 1,030.00	
2	$ 1,030.00	3.0%	$ 30.90		$ 1,060.90	
3	$ 1,060.90	3.0%	$ 31.83		$ 1,092.73	
4	$ 1,092.73	3.0%	$ 32.78		$ 1,125.51	
5	$ 1,125.51	3.0%	$ 33.77		$ 1,159.27	

Let's look at the interest earned each period.

Year 1: I(1) = .03*(1000) = 30.00
Year 2: I(2) = .03*(1000) +.03*(30) = 30.00 + .90 =30.90
Year 3: I(3) = .03*(1000) +.03*(30) +.03*(30.90) = 30.00 + .90 +.927 = 31.827
Year 4: I(4) = .03*(1000) +.03*(30) +.03*(30.90) + .03*(31.827)
= 30.00 + .90 +.927 + .95481 = 32.78181
Year 5: I(5) = .03*(1000) +.03*(30) +.03*(30.90) + .03*(31.827) +.03*(32.78181)
= 30.00 + .90 +.927 + .95481 + .983454 = 33.765264

I've shown all the details so that you can fully see and understand the internal process of compound interest. But now we will employ mathematics to make our life as financial managers a little bit easier. Let i be the letter name we assign to the rate of interest. In the example above, $i = .03$. Let P be the letter name we assign to the original deposit (P is from the word Principal). Now we restate the Basic Example in mathematical language.

Year	Interest Rate	Beginning Balance	Interest Earned	Ending Balance
1	i	P	$i \times P$	$P + i \times P = (1+i)P$
2	i	$(1+i)^1 P$	$i \times (1+i)^1 P$	$(1+i)P + i \times (1+i)P = (1+i)^2 P$
3	i	$(1+i)^2 P$	$i \times (1+i)^2 P$	$(1+i)^2 P + i \times (1+i)^2 P = (1+i)^3 P$
4	i	$(1+i)^3 P$	$i \times (1+i)^3 P$	$(1+i)^3 P + i \times (1+i)^3 P = (1+i)^4 P$
5	i	$(1+i)^4 P$	$i \times (1+i)^4 P$	$(1+i)^4 P + i \times (1+i)^4 P = (1+i)^5 P$
t	i			$(1+i)^t P$

Note how easy life is now. We need the time that interest is being paid, we need the initial deposit (the initial principal amount), and the interest rate. We can then compute the future value (FV) at any future point in time, using the equation FV = $(1+i)^t P$. For example, if \$100 is deposited at the bank for 8 years, at a 7% interest rate, the amount of the ending balance is computed as follows: FV = $(1+.07)^8 \times 100 = \$171.82$.

Do the following practice problems to cement these ideas in your mind.

1. What is the bank balance after 3 years, if the interest rate is 4%, and the initial deposit is \$500? (Answer: \$ 562.43)
2. After 5 and a half years? (Answer: \$ 620.37)
3. Now suppose the interest rate is 4.5%. What will a \$700 deposit be worth in 7 years? (Answer: \$ 952.60)

Seems pretty straightforward, right? You should make sure these concepts are perfectly clear to you, since we will be building our whole structure of financial mathematics on this basic understanding.

But there are some quirks ahead. In the next set of notes we will take up the problems that arise when interest is compounded on a schedule other than annually. For example, a bank may compound interest on a quarterly, or monthly basis. How do we handle that?