UNIT 3 LECTURE: z-Scores

Moving forward in the course assumes that you have mastered the information which has been covered to this point. You must master the information in each Unit in order to succeed in the next. At this point (before going forward), you should understand that distributions (sets of data) can be fully described by the following three things: (1) a measure of central tendency (mean, median, or mode), (2) a measure of variability (standard deviation, σ or s), and (3) a shape (symmetrical or skewed). Further, you should be able to calculate the measures of central tendency and the measures of variability. If you feel confident in your understanding of the aforementioned, then you are ready to move forward. If you do not feel confident, then consider reviewing previous lectures before moving forward.

Z-scores

Unit 3 focuses on Z-scores. Calculating a Z-score is a descriptive statistical method that allows you to use the mean and the standard deviation from a set of data to find the exact **LOCATION** of any **one** of the X- scores in that set of data . Let me begin by attempting to get across to you what the general idea of a z-Score is.

It may be helpful to keep in mind that an X-score is any score in a data set. For example, imagine that you took an exam in a course, the professor would have a data set: all of the students' scores. Each exam score would be an X-score, but, if you wanted to know the LOCATION of your exam score (where **your** score is located in the data set of the class exam scores), you would need the mean of the class exam scores and the standard deviation of the class exams scores, which would allow you to calculate **a z-score**.

A z-score is a "signed" number. That is, a z-score has a number and a sign. And, this "signed number," or z-score, gives you 2 pieces of information that allow you to locate ANY X-score on a distribution or data set.

The Two Pieces of Information Provided by a Z-score

1. The **sign** of the z-score is either positive (+) or negative (-)
   If the sign of the z-score is positive, then you would know that the location of the X-score is ABOVE mean.
   If the sign of the z-score is negative, then you would know that the location of the X-score is BELOW the mean.

2. The **numerical** value of the z-score tells you the distance from the mean at which the X-score is located, in units of **standard deviations**.

This Next Part is Really Important ☺

Imagine that a professor returned an exam to you, and instead of giving you a score with which you are very familiar, like a 70 or an 85, the professor gave back your exam with a grade in the form of a z-score. Read that previous sentence again.

So, there you are looking at your exam, and your grade is: **Z = 2**. How did you do on the exam? What is your exam grade? Let me remind you of the two pieces of information that your Z-score gives you: The first piece of information is that your Z-score is positive, so you know that your X-score (your exam score) is ABOVE the mean. You know your exam score (X-score) is located ABOVE the mean, which might make you happy.

But, there is also another important question: **How much above the average** (or mean) is your X-score (your exam score)? The second piece of information that a z-score gives you provides the answer. Remember that the second piece of information that your z-score =2 is the numerical value. And you might remember that the numerical value tells you the distance from the mean in units of standard deviations. Therefore, you KNOW that, based on your z-score (z = 2), your exam score (X-score) is LOCATED a full TWO standard deviations above the mean ☺ CONSIDER reading this section over once or twice because if you really get this part, the rest will be a lot easier.

Now, still based on the previous section, imagine that the professor told you that the class mean (**M**) was 60 and that the standard deviation **s** =5. Now, you can figure out exactly what your exam score is.

How to Figure it Out

Well, you know your z-score is z =2. And, as we discussed above, you would know that your exam score is 2 standard deviations above the mean. If the mean equals 60 (M = 60), and each standard deviation is equal to 5 points because (s = 5), then you KNOW that your X-score (your exam score) is 70! How do you know?

Because your z-score tells you that your exam score is LOCATED 2 standard deviations above the mean (your z = 2). If each standard deviation is 5 points, and you are above by 2, then you are above by 10 points (2 standard deviations, 5+5). So, if the mean is 60 and you are 2 standard deviations (10 points because each standard deviation is 5 points), then your exam score (your X-score) MUST be 70!!

Transforming Z scores to X-scores

Transforming a Z-score to an X-score is exactly what we just did above ☺ Let's do another example: Imagine a distribution of exam scores with a mean =80 and a standard deviation =4 (M =80, s = 4).

You get back your exam and your grade is in the form of a Z-score, and it is Z= -1. Notice the Z is a negative. Can you figure out what your X-score is?

If your Z-score is -1, then you know your X-score is BELOW the mean. You also know that it is below the mean by 1 standard deviation. Because the mean is 80 (M=80) and the standard deviation is equal to 4 (s = 4), you know that your X-score is BELOW the mean by 4 points. Thus, your X-score = 76.

Another example:

If your Z-score is 1.5 (z = 1.5) for the same distribution (mean=80 and standard deviation =4), what is your corresponding X-score? You know the X-score is above the mean because the Z-score is positive and from the numerical value of the Z-score, 1.5, you know it is above the mean by one and half standard deviations. The standard deviation is 4 (s = 4); therefore, one and a half, 1.5, standard deviations equals 6. Your X-score is 86. Can you see that?

What was done above, was done "in our heads," which is a good way to come to understand what a Z-score is and how to use it. Doing it in "our heads" is usually easy (after lots of practice) when we are working with whole numbers, but the calculations become cumbersome when the numbers are not psychologically comfortable.

For example, for a distribution with a mean $\mu = 80$ and a standard deviation of 2 ($\sigma=2$) ….remember that σ just means the data come from a population; whereas (s) just means that the data come from a sample, but both (s) and (σ) are symbols for standard deviation)….also remember that M is the mean of a sample and μ is the mean of a population (as described in units 1 and 2)

So let's repeat: You have a distribution with a $\mu=80$ and a $\sigma = 2$. The question is: What is the X-score that corresponds to a Z=1.28?

How to Transform Z-scores to X-scores Using a Formula

The logical progression to get the answer "in your head" is the same, but the numbers make it difficult to do it in your head. Therefore, you can use the following formula:

$$X = \mu + Z\sigma$$

If you used the formula, then you would get the following: $X = 80 + (1.28)(2)$

$$X = 80 + 2.56$$

$$X = 82.56$$

If you find it cumbersome to do it in your head, you can always use the formula.

Transforming X- scores to Z- scores (In your head)

Much like we transformed Z-scores to X-scores (all that we did above), we can do it in reverse; that is, transform X-scores to Z-scores. This time, we start out with an X-score and we want to find its corresponding Z-score.

Imagine that the professor returned an exam with a grade of 65. Further, the professor told you that the mean for the class exams was M =50 and the standard deviation was 10 (s = 10). If your X-score is 65, what is your Z-score? Try to think it through before going on to read the answer.

To get to the answer, first locate the mean, which you know is 50. Your score is 65. Therefore you know that your Z-score is going to be positive because your 65 is above the mean. You also know that your score of 65 is 15 points above the mean. So…ask yourself: How many standard deviations above the mean is my score? Well…if one standard deviation is equal to 10 points, can you see that you are 1.5 (one and a half) standard deviations above the mean? And your z-score would be 1.5? It is really worth spending a little time here until you can **"see"** this.

How to Transform X-scores to Z-scores (Using a Formula)

If you understand what has been discussed to this point, then you can calculate Z-scores in your head when the numbers are easy to work with. But, sometimes, the numbers are not so easy to work with. For example, for a distribution with a M=50 and s =10. What is the Z-score that corresponds to an X= 43?

How to figure this out is exactly the same as with the other examples, but this time, your score is 7 points below the mean and the s =10, so it is simply harder to calculate exactly how many standard deviations X=43 is below the mean in your head. When the numbers are difficult to work with, then simply use the following formula:

$$z = \frac{X - M}{s}$$

If you used the formula, to figure out what the Z-score is for X= 43, it would look like this:

$$z = \frac{43-50}{10}$$

$z = -0.7$ (notice it is a negative z)

Using z-Scores to Standardize a Distribution (Last Section)

Imagine that you got an exam score in biology of X = 50 and an exam score in chemistry of X = 50. In which class did you get the better grade?

You may say that you did equally well, but the reason you may say that is that you are LOCATING your exam scores of 50 on the same 0-100 scale used throughout almost all of your educational years.

However, to know in which class you have the better standing, you can't directly compare your biology X = 50 score to your chemistry X= 50 score because those scores are on TWO different distributions, which are defined by two different measure of central tendency and variability.

Just like you cannot compare apples to oranges, you cannot compare two unlike distributions. In order to make a comparison, you must first find a way to compare apples to apples and oranges to oranges. That is, you MUST make the different distributions both apples or both oranges.

How can we go about changing two different X-score distributions (biology/apples and chemistry/oranges) to the same "fruit" (or distribution) so that we can make comparisons between them?

When you change X-scores to Z-scores a cool thing happens: the mean and standard deviation becomes the same because Z-score distributions are standardized. Therefore, Z-score distributions always have a mean=0 and a standard deviation =1. Therefore, when you change the Chemistry grade (X-score) to a Z-score and the Biology grade to a Z-score, both grades will now be located on a Z-distribution with, again, a mean = 0 and a standard deviation = 1, and we can now compare your Chemistry grade to your Biology grade, despite the fact that those grades come from two different distributions.

Reiterating, when you transform an X-distribution to a Z-distribution, you change the different means of the X-distributions to the same mean (M=0) and you change the different standard deviation of the X-distributions to the same standard deviation (s = 1). The Z-distribution is standardized such that the M will always be 0 (M=0) and the standard deviation will always be 1 (s= 1).

Let's Return to your Chemistry and Biology Grade:

You got a 50 on your chemistry test. The mean for the class was M = 55 and a standard deviation equals to 5 ($\sigma = 5$). You also got a 50 on your biology exam. The mean for the class was M=55 and $\sigma = 2$. On which test did you have a better class standing?

Can you figure out the answer at this point? If you can, then you have really understood all that has been said so far. If you cannot, then keep reading to find the answer, but you may want to review all that has been stated in this lecture up to this point.

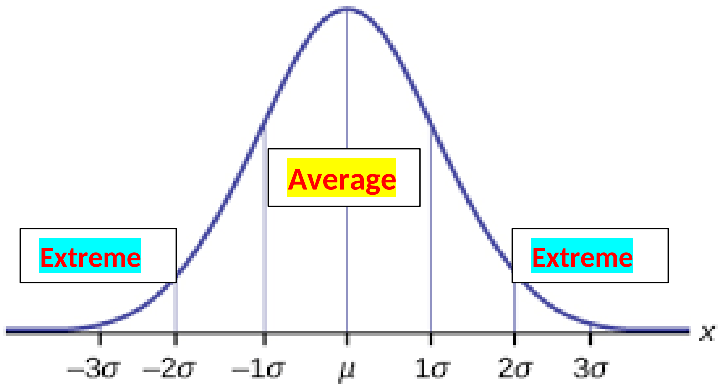
Let's get back to the example. Notice that you got the same score, X=50, on both exams. Also notice that the means for both exam distributions are the same (M =55), but the distributions have different standard deviations. Therefore, you can't compare the distributions the way they are because one is an "apple" and one is an "orange." But, if we changed both your X-scores (Biology X and Chemistry X) to Z-scores, then we can compare them because, remember, the Z-distribution is standardized with a M=0 and a standard deviation =1.

You can use the Z-formula to do the calculations, but you can also try to do the calculations in your head for your relative class standing.

You got an X=50 in Chemistry in a distribution with a mean =55 and $\sigma = 5$. Because your Chemistry score is below the mean, your Z score will be negative. Because you are 5 points below the mean, you are exactly 1 standard deviation below the mean; therefore, your Chemistry z-score is Z = -1.

In Biology, you got an X= 50 in a distribution with mean = 55 and $\sigma = 2$. Because your Biology score is below the mean, your Z score will be negative. But this time, because the $\sigma = 2$, you are 2.5 standard deviations below the mean; therefore, your Biology Z-score is -2.5. When we compare your Z= -1 in Chemistry to your Z= -2.5 in Biology, we can see that your Chemistry standing is much, much higher than your Biology standing, despite having gotten the same score. Your Chemistry score would be considered an "average" score because it is within the average range of a normal distribution, but your Biology score is a true failure because it is located more than 2 standard deviations below the mean. I understand that, at this point, the previous sentence probably doesn't make too much sense. So, let me explain it.

The ideas of "central/average" scores and "extreme" scores are an important concept.



Look at the above distribution. Notice that it is a symmetrical distribution with the mean (μ) at the center. Also note that 1σ, 2σ, and 3σ written along the same bottom line as the mean indicate distance from the mean in standard deviations, in essence, they are z-scores.

It is important to know that scores located within 1 standard deviation of the mean are referred to as "central" or "average" scores. And scores located at or more than 2 standard deviations away from the mean are referred to as extreme scores.

Your Chemistry Z-score of z = -1 falls exactly on the line to make it an "average" score. But your Biology Z-score of z = -2.5 makes it an extreme score.

Z-distributions are used all over the place in the "real world," including many of the standardized tests in our field of Psychology. For example, imagine that you took an IQ test. Further, know that the mean for adult IQ is M = 100 with a σ = 15. Can you figure out the range of "average" IQ scores?

The answer is 85-115 because if the mean is 100, then one standard deviation (15) below 100 is 85, and one standard deviation above the mean of 100 is 115. Therefore, the range of scores that are located within the average in a normal distribution is from 85 to 115.

Can you guess how high your IQ would have to be in order to be considered EXTREMELY intelligent?

Well, extreme scores are scores that are located at or above 2 standard deviations from the mean. If the mean is 100, then two standard deviations away from the mean would be 130 (each standard deviation is equal to 15). Get it? You would have to score at least a 130 to get an extremely high IQ score.

If you scored an 90 on the IQ test, would your score be “average” or “extreme”? It would be average because it falls within the “average” range of 85-115. What if you scored a 70? In this case you would have an extreme score and an extremely low IQ because your score of 70 is located at 2 standard deviations below the mean.

At this point, we are finished with Z-scores with some introduction to the idea of the normal distribution. Now, let us turn directly to the “normal distribution”:

The normal distribution has some defining characteristics:

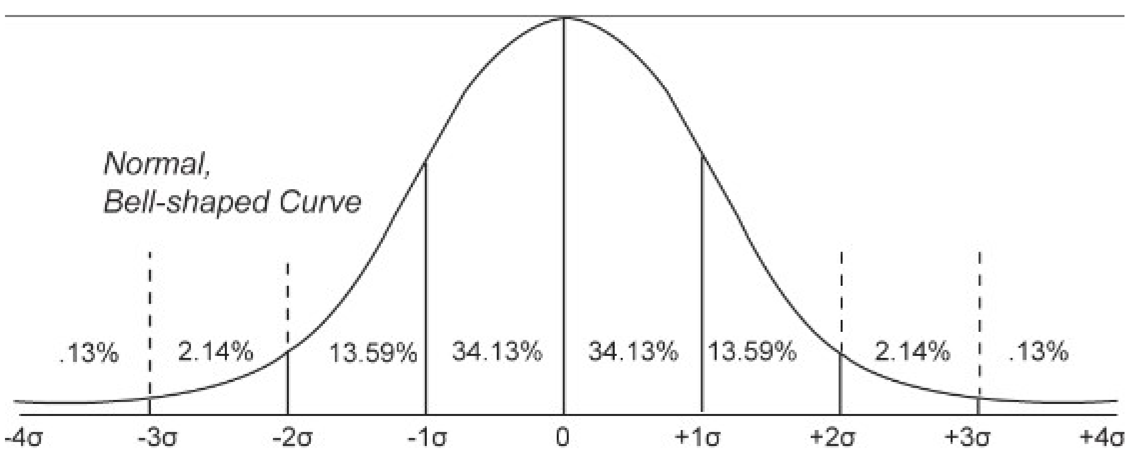
1. It is symmetrical in shape

2. All measures of central tendency are located dead center and all have the same value

3. There are more scores located near the mean (the center of the distribution…that “average” range) than there are scores as we move away from the mean

And, importantly,

4. A distribution is normal if and only if it has specific proportions as seen in this image:



<……………………70 85 μ=100 115 130--------------------- >

This above is the bell-shaped, normal curve. This is the distribution on which, for example, standardized IQ test, SAT scores, GRE scores, and other standardized scores are based. In our earlier example of IQ, notice that 68.26% of the population have “average” IQs because “average” means within one standard deviation from the mean. In the case of IQ, with a M=100 and $\sigma = 15$, I hope you can see that “average” IQ scores are located between one standard deviation below the mean and one standard deviation above the mean (highlighted in yellow). Moreover, notice that only about 2.28 (2.27) of the population have IQ scores that are extremely high. Remember that extreme scores are those that are located at or above 2 standard deviations from the mean (highlighted in green).

So, if I asked you, what percentage of the population has average IQ scores”, what would be your answer? The answer would be 68.13%

What if I asked you, what percentage of the population has an IQ score that is considered extremely low? The answer would be approximately 2.28% (2.27, according to the graph)

What if I asked you, what is the probability of selecting an average IQ from this population? The answer would be 68.13%

What is the probability of selecting an extremely low score? 2.28%

What is the probability of selecting an extremely high score? 2.28%

What is the probability of selecting a score lower than the mean? 50%....remember the distribution is symmetrical so 50% of the scores are above the mean and 50% are below the mean.