Homework 10

2) Find $\frac{dx}{dt}$ at $x=-2$ and $y=2x^2+1$ if $\frac{dy}{dt}=-1$

$$\frac{d}{dt}[y] = \frac{d}{dt}[2x^2+1]$$

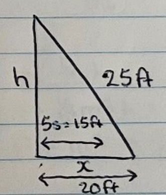
$$\frac{dy}{dt} = 4x \cdot \frac{dx}{dt}$$

$$-1 = 4(-2)\frac{dx}{dt}$$

$$\frac{dx}{dt} = \frac{-1}{-8}$$

$$\frac{dx}{dt} = \frac{1}{8}$$

6)



$\frac{dx}{dt} = -1$ ft/sec

$x=15$, $\frac{dh}{dt} = ?$

$$25^2 = 15^2 + h^2$$

$$h = \sqrt{400}$$

$$h = 20$$

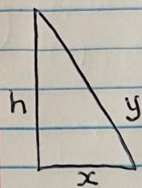
$$\frac{d}{dt}[x^2+h^2] = \frac{d}{dt}[25^2]$$

$$2x \cdot \frac{dx}{dt} + 2h \cdot \frac{dh}{dt} = 0$$

$$\frac{2h \cdot \frac{dh}{dt}}{2h} = \frac{-2x \cdot \frac{dx}{dt}}{2h}$$

$$\frac{dh}{dt} = \frac{-x \cdot \frac{dx}{dt}}{h} = \frac{-15 \cdot -1}{20} = \frac{15}{20} = \frac{3}{4} \text{ ft/sec.} = 0.75 \text{ ft/s.}$$

8.



$\frac{dx}{dt} = 16\text{mph} \quad , \quad x = 4$

$\frac{dh}{dt} = 12\text{mph} \quad , \quad \cancel{h=3}$

$\frac{dy}{dt} = ?$

Find t : $v = \frac{d}{t} \quad \therefore \quad 16 = \frac{4}{t} \quad \therefore \quad t = \frac{4}{16} = \frac{1}{4}$ seconds

Find h : $v = \frac{h}{t} \therefore \quad 12 = \frac{h}{1/4} \quad \therefore h = 12 \cdot \frac{1}{4} = 3$ miles

Find y ;

$$y^2 = x^2 + h^2$$
$$y = \sqrt{25}$$
$$y = 5$$

Solve :

$$\frac{d}{dt}[y^2] = \frac{d}{dt}[x^2 + h^2]$$

$$2y\frac{dy}{dt} = 2x \cdot \frac{dx}{dt} + 2h\frac{dh}{dt}$$

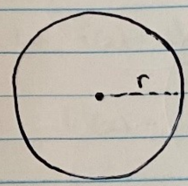
$$\frac{dy}{dt} = \frac{2x \cdot \frac{dx}{dt} + 2h \cdot \frac{dh}{dt}}{2y}$$

$$\frac{dy}{dt} = \frac{x \cdot \frac{dx}{dt} + h \cdot \frac{dh}{dt}}{y}$$

$$= \frac{(4)(16) + (3)(12)}{5}$$

$$\frac{dy}{dt} = 20\text{mph}$$

20)



$\frac{dr}{dt} = 1 m/s$, $r = 20m$

$V = \frac{4}{3}\pi r^3$

$\frac{d}{dt}[V] = \frac{d}{dt}[\,^4/_3\, \pi r^3]$

$\frac{dV}{dt} = 4\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(20)^2(1) = \underline{1600\pi} \; m^3/s = \underline{5026.55} \, m^3/s$

26) $r = \,^5/_{16}\, h$

$V = \,^1/_3\, \pi r^2 h = \frac{16}{15}\pi r^3$

$A = \pi r^2 \rightarrow \frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dV}{dt} = \frac{16}{5}\pi r^2 \frac{dr}{dt} \rightarrow \frac{dr}{dt} = \frac{5}{16\pi r^2}\frac{dV}{dt}$

$\therefore \frac{dr}{dt} = \frac{8}{25\pi}$ ft/min

$\therefore \frac{dA}{dt} = 2\pi\left(\frac{25}{8}\right)\left(\frac{8}{25\pi}\right) = 2 \; ft^2/min$

$= 2 \; ft^2/min$

$L(x)$ to $f(x) = x + x^4$ near $a=0$

50) $L(x) = f(a) + f'(a)(x-a)$

$$f'(x) = \frac{d}{dt}x + \frac{d}{dt}x^4$$
$$= 1 + 4x^3$$

$$L(x) = f(a) + f'(a)(x-a)$$
$$= f(0) + f'(0)(x-0)$$
$$= 0 + 1 \cdot (x-0)$$
$$= x$$
$$L(x) = x.$$

58) let $f(x) = \cos x$ and $a=0$. $\frac{d}{dt}$:

$$f'(x) = -\sin x$$
$$\therefore L(x) = f(a) + f'(a)(x-a)$$
$$= f(0) + f'(0)(x-0)$$
$$= 1 + 0 \cdot (x-0)$$
$$= 1$$

$f(0.03)$ within 0.01 is the

$L(0.03) = 1.00$

60) Let $f(x) = \frac{1}{x}$ and $a=1$

$f'(x) = -\frac{1}{x^2}$

$L(x) = f(a) + f'(a)(x-a)$
$= f(1) + f'(1)(x-1)$
$= 1 + (-1)\cdot(x-1)$
$= -x + 2$

$L(0.98) = -(0.98) + 2$
$= 1.02$

78) dV if the sides of a cube change from 10 to 10.1.

$V = x^3$

$\therefore \frac{dV}{dx} = 3x^2$

$\therefore dV = (3x^2)\,dx$

$x = 10$, $dx = 10.1 - 10 = 0.1$ then

$dv = 3(10)^2(0.1)$

$dv = 30$

84) $dr = \pm 0.1$

$r = 5$

$V = \frac{4}{3}\pi r^3$

$\therefore\ dv = 4\pi r^2\, dr$

$= 4\pi(5)^2(0.1)$

$= 10\pi$

$dv = \pm 10\pi \ / \ \pm 31.42$

Relative error

$$\frac{dv}{V} = \frac{4\pi r^2 dr}{4/3\,\pi r^3} = \frac{dr}{1/3\, r} = \frac{3dr}{r} = \frac{3(\pm 0.1)}{5}$$

$$= \pm 0.06$$