3.1 Functions

A ~~function~~ relation is a set of pairs of input and output values. You can write them as ordered pairs.

The domain of a relation is the set of all inputs, or x-coordinates of the ordered pairs.

The range of a relation is the set of all outputs, or y-coordinates of the ordered pairs.

A function is a relation in which each element of the domain is paired with exactly one element in the range.

Look for: The same *x*-value, but different *y*-values! This means it is <u>not a function</u>!

Are the following relations function? State why or why not! Then state the domain and range!

a) $\{(5,1),(-3,5),(8,1),(2,-7),\}$

Function: Yes

Domain: $\{-3, 2, 5, 8\}$

Range: $\{-7, 1, 5\}$

b) $\{(3,1),(-2,4),(3,3),(1,0),\}$

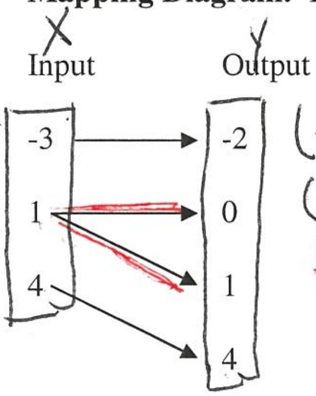
write in order from least to greatest

Function: No $x=3$ repeats

Domain: $\{-2, 1, 3\}$

Range: $\{0, 1, 3, 4\}$

Mapping Diagram: Identify the domain and range. Then tell whether the relation is a function.

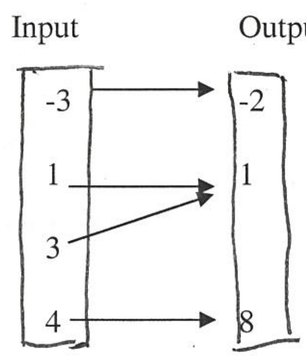


(-3, -2)
(1, 0)
(1, 1)
(4, 4)

Function: No $x=1$ repeats

Domain: $\{-3, 1, 4\}$

Range: $\{-2, 0, 1, 4\}$



(1, 1)
(3, 1)

Function: Yes

Domain: $\{-3, 1, 3, 4\}$

Range: $\{-2, 1, 8\}$

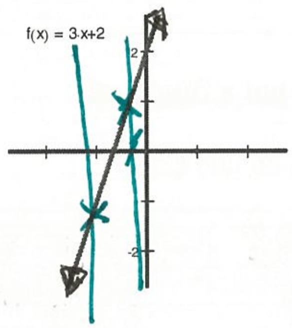
Vertical Line test: A relation is a function if and only if no vertical line intersects the graph of the relation at more than one point. If a vertical line passes through two or more points on the graph, then the relations is *not* a function.

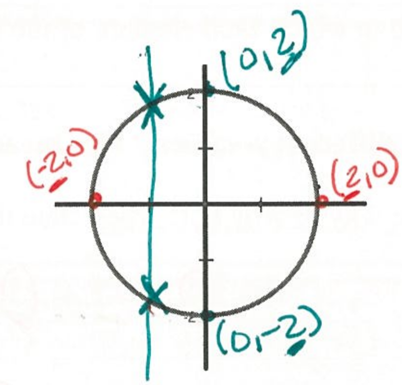
For a graph to be a function, all vertical lines must touch in only 1 spot (or not touch at all)

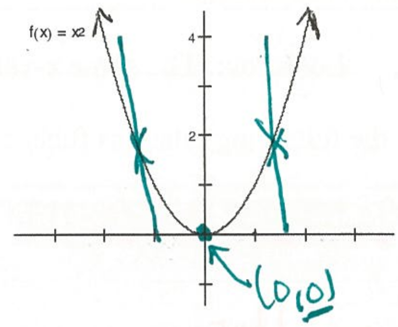
For the domain, scan your eyes from furthest left to furthest right.

For the range, scan your eyes from the lowest point to the highest point.

Using the Vertical Line Test, determine whether each relation is a function. Then state the domain & range.



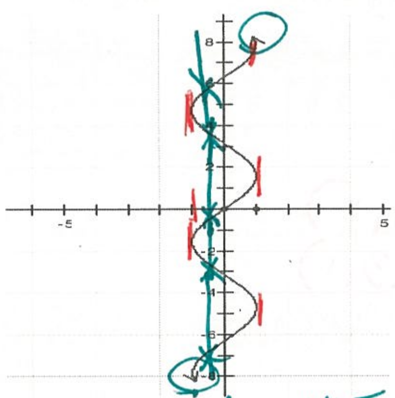




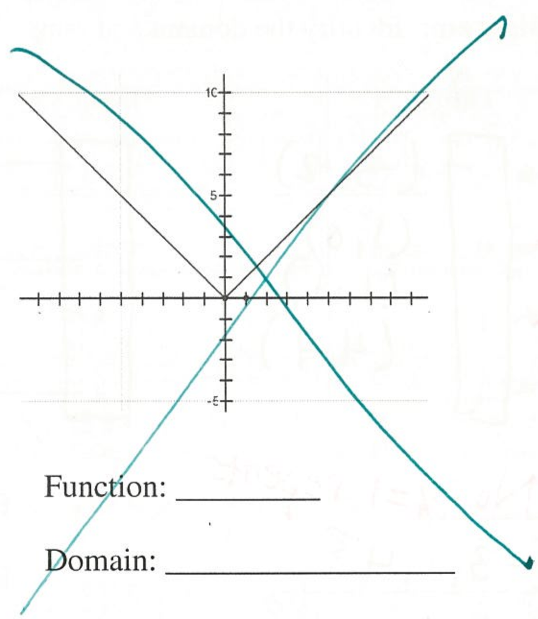
Function: Yes
Domain: $\mathbb{R}$ all real numbers
Range: $\mathbb{R}$ all real numbers

Function: No fails VLT
Domain: $-2 \le x \le 2$
Range: $-2 \le y \le 2$

Function: Yes
Domain: $\mathbb{R}$ all real numbers
Range: $y \ge 0$



Function: No fails VLT
Domain: $-1 \le x \le 1$
Range: $\mathbb{R}$ all real numbers



Function: ______
Domain: ______
Range: ______

Earnings Per Hour				
Name	Bob	Dave	Jane	Sam
y Pay	$9	$11	$5	$15

Function: Yes

Domain: {Bob, Dave, Jane, Sam}

Range: {5, 9, 11, 15}

Test Scores				
Name	Jim	James	Jorge	James
Score	65	91	94	88

Function: No x=James repeats

Domain: {James, Jim, Jorge}

Range: {65, 88, 91, 94}

Finding the Domain of a Function Algebraically

Two rules to remember:

1. The denominator of a fraction cannot equal zero.
2. The number under a square root cannot be negative.

Algebraically determine the following domains.

the set of

1. $d(y) = y + 3$ $\mathbb{R}$ all real numbers $\{x \in \mathbb{R}\}$

such that

2. $b(n) = \sqrt{2n - 8}$ $\rightarrow$ $2n - 8 > 0$; $+8 \quad +8$; $\frac{2n}{2} > \frac{8}{2}$; $n > 4$; $\{n \mid n > 4\}$

3. $u(x) = \dfrac{x-5}{2x+4}$ $2x + 4 = 0$; $-4 \quad -4$; $\frac{2x}{2} = \frac{-4}{2}$; $x = -2$; $\{x \mid x \neq -2\}$

4. $y(c) = \dfrac{2}{c^2 + 3c}$

$c^2 + 3c = 0$

Factor by GCF

$c(c + 3) = 0$

$\boxed{c = 0}$ $c + 3 = 0$; $-3 \quad -3$; $\boxed{c = -3}$

$\{c \mid c \neq -3, 0\}$

OR

$\{c \mid c \neq -3, c \neq 0\}$

Function Notation:

$y = 3x + 2$ can be written in a "fancy notation" → $f(x) = 3x + 2$.

$f(x)$ is read as f of x and does NOT mean *f* times *x*.

"Normal Way"	Function Way
(1,5)	$f(1) = 5$ ← output input
Simplify $y = 2x - 1$ when $x = 3$ Plug in 3 for x and evaluate	$f(x) = 2x - 1$; find $f(3)$

Given $f(x) = 3x - 2$ and $g(x) = x^2 + 3$, find the following:

a) $f(2) = 3(2) - 2$
$= 6 - 2$
$= 4$

b) $g(-3) = (-3)^2 + 3$
$= 9 + 3$
$= 12$

c) $f(4)$

d) $g(5)$

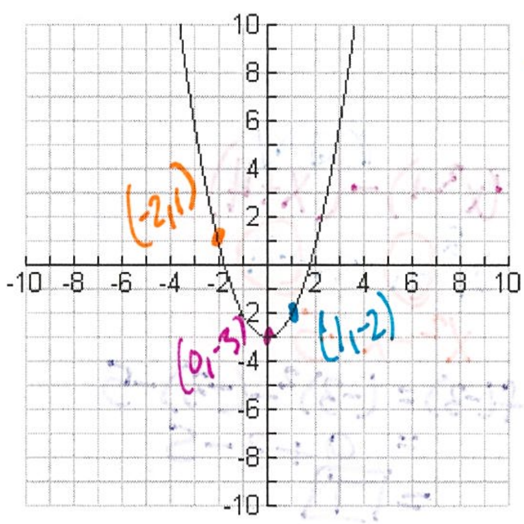
e) $f(\frac{1}{2}) = 3(\frac{1}{2}) - 2$
$= 1.5 - 2$
$= -0.5$ or $-\frac{1}{2}$

f) * $g(f(3))$

work inside out
$f(3) = 3(3) - 2$
$= 9 - 2$
$= 7$
$g(7) = (7)^2 + 3$
$= 49 + 3$
$= \boxed{52}$

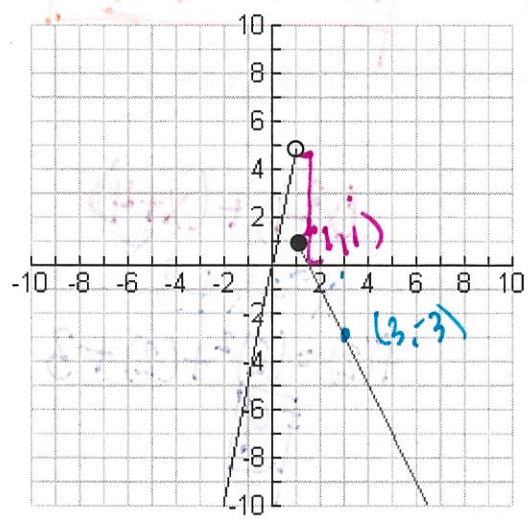
What is what y when x = ?

Find $f(1)$, $f(-2)$, $f(0)$.



$f(1) = -2$
$f(-2) = 1$
$f(0) = -3$

Find $g(1)$, $g(3)$, $g(-5)$.



$g(1) = 1$
$g(3) = -3$
$g(-5) = \varnothing$

Arithmetic Combinations of Functions

> **Sum, Difference, Product, and Quotient of Functions**
>
> Let f and g be two functions with overlapping domains. Then, for all x common to both domains, the *sum, difference, product,* and *quotient* of f and g are defined as follows.
>
> **1.** Sum: $(f+g)(x) = f(x) + g(x)$
>
> **2.** Difference: $(f-g)(x) = f(x) - g(x)$
>
> **3.** Product: $(fg)(x) = f(x) \cdot g(x)$
>
> **4.** Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$

EXAMPLE

$f(x) = 2x - 3$ and $g(x) = x^2 - 1$

Find

a) $(f+g)(x)$ $(2x-3)+(x^2-1)$

$\boxed{x^2+2x-4}$

b) $(f-g)(x)$ $(2x-3)-(x^2-1)$

$2x-3-x^2+1$

$\boxed{-x^2+2x-2}$

c) $(fg)(x)$ $(2x-3)(x^2-1)$

$2x^3-2x-3x^2+3$

$\boxed{2x^3-3x^2-2x+3}$

d) $\left(\frac{f}{g}\right)(x)$ $\boxed{\frac{2x-3}{x^2-1}}$

Evaluating Arithmetic Combinations of Functions

EXAMPLE

$f(x) = x^2 - 1$ and $g(x) = x + 4$

Find

a) $(f+g)(2)$

$(x^2-1)+(x+4)$

x^2+x+3

$f(2)=(2)^2+(2)+3$

$=\boxed{9}$

b) $(f-g)(-3)$

$(x^2-1)-(x+4)$

$x^2-1-x-4$

x^2-x-5

$f(-3)=(-3)^2-(-3)-5$

$=9+3-5$

$=\boxed{7}$

d) $\left(\frac{f}{g}\right)(7)$

d) $(fg)(0)+f(6)$

$(fg)=(x^2-1)(x+4)$

x^3+4x^2-x-4

$(fg)(0)=0^3+4(0)^2-(0)-4$

$=\boxed{-4}$

$f(6)=(6)^2-1$

$=36-1$

$=\boxed{35}$

$(fg)(0)+f(6)=-4+35=\boxed{31}$