Shakendra Alston – EE330 – Test 3

1) a) $x(t) = 10e^{-2(t-2)}u(t-2)$

$\mathcal{L}[x(t)] \longleftrightarrow X(s)$

where $e^{-2t}u(t) \xleftrightarrow{L.T} \frac{1}{s+2}$

$\therefore \frac{-at}{e}u(t) = \frac{1}{s+a}$ from standard

Let say $y(t) = e^{-2t}u(t)$

$\therefore x(t) = 10\,y(t-2)$

$X(s) = 10e^{-2s}Y(s)$

where $y(t) \xleftrightarrow{L.T} Y(s)$

$\therefore Y(s) = \frac{1}{s+2}$

from shifting property

$y(t-a) \xleftrightarrow{LT} e^{-as}Y(s)$

$\therefore \mathcal{L}[x(t)] = \frac{10 \cdot e^{-2s}}{s+2} = X(s)$

b) $x(t) = \sin 2(t-1)\,u(t-1)$

where $y(t) = \sin 2t\,u(t)$

where $Y(s) = \frac{2}{s^2+4}$

$\therefore x(t) = y(t-1)$

$x(t) \xleftrightarrow{LT} X(s)$

from shifting property

$X(s) = e^{-s(1)}Y(s)$

$X(s) = e^{-s}\frac{(2)}{s^2+4}$

$\therefore y(t) \xleftrightarrow{LT} Y(s)$

$\because \sin(at) \xleftrightarrow{LT} \frac{a}{s^2+a^2}$ from standard

2) a) $Y(s) = \frac{1}{s(s-2)} = \frac{-2}{-2s(s-2)} = -0.5\left[\frac{s-s-2}{s(s-2)}\right]$

$= -0.5\left[\frac{s-2-s}{s(s-2)}\right] = -0.5\left[\frac{s-2}{s(s-2)} - \frac{s}{s(s-2)}\right]$

$= \frac{-0.5}{s} + \frac{0.5}{s-2} = \frac{0.5}{s-2} - \frac{0.5}{s}$

$\mathcal{L}^{-1}\left[\frac{1}{s}\right] = u(t)$

$\mathcal{L}^{-1}\frac{1}{s-a} = e^{at}u(t)$

by inverse laplace: $x(t) = 0.5e^{2t}u(t) - 0.5\,u(t)$

$\therefore x(t) = 0.5\left[e^{2t} - 1\right]u(t)$

b) $X(s) = \frac{2}{s^2+s+1} = \frac{2}{s^2+2(0.5)s+0.25+0.75}$

$= \frac{2}{s^2+2(0.5)s+(0.5)^2+(0.866)^2} = Y(s) = \frac{2}{(s+0.5)^2+(0.866)^2}$

$\mathcal{L}^{-1}\frac{1}{(s+a)^2+b^2} = \frac{e^{-at}\sin bt}{b}u(t)$ by inverse laplace

$\therefore x(t) = \frac{2}{0.866} \cdot e^{-0.5t}\sin(0.866t)\,u(t)$

$\therefore x(t) = 2.31 e^{-0.5t}\sin(0.866t)\,u(t)$

3) $\frac{dy(t)}{dt} + 5y = 3u(t)$; $y(0) = 1$ $\quad \frac{dy(t)}{dt}$ $sy(s) - y(0)$

$\Rightarrow sy(s) - y(0) + 5y(s) = 3 \cdot \frac{1}{s}$

$\Rightarrow sy(s) - 1 + 5y(s) = \frac{3}{s}$

$y(s)(s+5) = \frac{3}{s} + 1$

So laplace domain is $Y(s) = \frac{(s+3)}{s(s+5)} = \frac{A}{s} + \frac{B}{(s+5)}$

$\Rightarrow s+3 = \frac{As(s+5)}{s} + \frac{Bs(s+5)}{(s+5)}$

$s+3 = A(s+5) + Bs$

$s+3 = As+5A+Bs$

$s+3 = (A+B)s+5A$

$A+B=1$

$5A=3$

$A = \frac{3}{5}$

$\frac{3}{5}+B=1 \quad B=\frac{2}{5}$

$Y(s) = \frac{(s+3)}{s(s+5)} = \frac{3/5}{s} + \frac{2/5}{(s+5)}$

$\Rightarrow Y(s) = \frac{3}{5}\cdot\frac{1}{s} + \frac{2}{5}\cdot\frac{1}{(s+5)} = \frac{1}{5}\left[\frac{3}{s} + \frac{2}{(s+5)}\right]$

$Y(s) = \frac{1}{5}\left[\frac{3}{s} + \frac{2}{(s+5)}\right]$; $\boxed{y(t) = \frac{1}{5}\left[3u(t) + 2e^{-5t}u(t)\right]}$

4) $C = \begin{matrix} 1 & 1 \\ 1 & 2 \end{matrix}$ $= 1+2 = 3$

$\det C = \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 2-1 = 1$

let x, y as eigenvalues

$x + y = 3$ — ①

$xy = 1$

$(x-y)^2 = (x+y)^2 - 4xy$

$(x-y)^2 = 9 - 4xy = 5$

$x - y = 2.236$ — ②

$x + y = 3$

$x - y = 2.236$

$\frac{2x}{2} = \frac{5.236}{2}$

$x = 2.618$

$y = 3 - x$

$= 3 - 2.618 = 0.382$

5) $\frac{d^2(y(t))}{dt^2} + 9\frac{dy(t)}{dt} + 6y(t) = x(t)$

assume $y(0) = y_{01}$

$\frac{dy}{dt}(0) = y_{02}$

$\frac{d^2y}{dt^2}(0) = y_{03}$

$z_1 = y(t)$

$z_2 = \frac{d}{dt} y(t)$

$\begin{cases} \dot{z}_1 = z_2 \\ \dot{z}_2 = 6z_1 - 9z_2 + x(t) \end{cases}$

$\dot{z} = Az + Bx$

where

$A = \begin{bmatrix} 0 & 1 \\ 6 & -9 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $\dot{z} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$ } $x = [x]$

$|sI - A| = \det \begin{bmatrix} s & -1 \\ -6 & s+9 \end{bmatrix} = s(s+9) - 6$

$= s^2 + 9s - 6$

$= 0$

$s_{1,2} = \frac{-9 \pm \sqrt{9^2 + 4 \cdot 6}}{2} < 0$

6) $\frac{Y(s)}{X(s)} = \frac{\frac{1}{Cs}}{sL + R + \frac{1}{Cs}}$ $(R = L = C = 1)$

$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + s + 1}$ $s = -1 \pm \sqrt{3}i$ (system is stable)

$\frac{X(jw)}{X(jw)} = \frac{1}{(-4 + 2j + 1)} = \frac{1}{\sqrt{9+4}} = \frac{1}{\sqrt{13}}$

$\theta = \tan^{-1}\left(-\frac{3}{2}\right) = \pi - \tan^{-1}\left(\frac{3}{2}\right) = 56.30$

$y(t) = x(t)(1 + H(jw))(\angle\theta) = \frac{10\cos(2t + 123.6)}{\sqrt{13}}$ (system is stable)