Kirk Ambrose
Due Oct, 5
EE 330

Signal and Systems Test 3

1. (a) $10e^{-2(t-2)}u(t-2)$

let $x(t) = 10e^{-2(t-2)}u(t-2)$

$\mathcal{L}[x(t)] \longleftrightarrow X(s)$

$e^{-2t}u(t) \xleftrightarrow{L.T} \dfrac{1}{s+2}$

$y(t) = e^{-2t}u(t)$

$x(t) = 10\,y(t-2)$

$X(s) = 10e^{-2s}Y(s)$

$y(t) \xleftrightarrow{L.T} Y(s)$

$Y(s) = \dfrac{1}{s+2}$

$\mathcal{L}[x(t)] = \dfrac{10 \cdot e^{-2s}}{s+2} = X(s)$

(b) $\sin(2(t-1))u(t-1)$

$x(t) = \sin 2(t-1)\,u(t-1)$

$y(t) = \sin 2t\,u(t)$

$y(t) \xleftrightarrow{L.T} Y(s)$

$Y(s) = \dfrac{2}{s^2+4}$

$x(t) = y(t-1)$

$x(t) \xleftrightarrow{LT} X(s)$

$X(s) = e^{-s(1)}Y(s)$

$X(s) = \dfrac{2e^{-s}}{s^2+4}$

2. (a) $X(s) = \dfrac{1}{s(s-2)} = \dfrac{-2}{-2s(s-2)} = -0.5\left[\dfrac{s-s-2}{s(s-2)}\right] = -0.5\left[\dfrac{s-2-s}{s(s-2)}\right]$

$= -0.5\left[\dfrac{s-2}{s(s-2)} - \dfrac{s}{s(s-2)}\right] = \dfrac{-0.5}{s} + \dfrac{0.5}{s-2} = \dfrac{0.5}{s-2} - \dfrac{0.5}{s}$

$x(t) = 0.5e^{2t}u(t) - 0.5u(t)$

$x(t) = 0.5\left[e^{2t}-1\right]u(t)$

(b) $X(s) = \dfrac{2}{s^2+s+1} = \dfrac{2}{s^2+2(0.5)s+0.25+0.75} = \dfrac{2}{s^2+2(0.5)s+0.25+(0.866)^2}$

$X(s) = \dfrac{2}{(s+0.5)^2+(0.866)^2}$

$x(t) = \dfrac{2}{0.866} \cdot e^{-0.5t}\sin(0.866t)u(t)$

$x(t) = 2.31e^{-0.5t}\sin(0.866t)u(t)$

3. $y(t)$ if $y(0)=1$ and $dy(t)/dt + 5y = 3u(t)$

$y(t) \Leftrightarrow Y(s)$

$u(t) \Leftrightarrow \dfrac{1}{s}$

$\dfrac{dy(t)}{dt} \Leftrightarrow sY(s) - y(0)$

$$\frac{dy(t)}{dt} + 5y = 3u(t)$$

$$sY(s) - y(0) + 5Y(s) = 3 \cdot \frac{1}{s}$$

$$sY(s) - 1 + 5Y(s) = \frac{3}{s}$$

$$Y(s)\left[s+5\right] = 3/s + 1 \Rightarrow Y(s)\left[s+5\right] = \frac{3+s}{s}$$

$$Y(s) = \frac{(s+3)}{s(s+5)}$$

$$\frac{(s+3)}{s(s+5)} = \frac{A}{s} + \frac{B}{(s+5)}$$

$$s+3 = \frac{As(s+5)}{s} + \frac{Bs(s+5)}{(s+5)}$$

$$s+3 = A(s+5) + Bs$$

$$s+3 = As + 5A + Bs$$

$$s+3 = (A+B)s + 5A$$

$$A+B=1$$

$$5A = 3$$

$$A = 3/5$$

$$A+B=1 \Rightarrow B = 1-A$$

$$= 1 - 3/5 = 5 - 3/5$$

$$B = 2/5$$

$$Y(s) = \frac{(s+3)}{s(s+5)} = \frac{3/5}{s} + \frac{2/5}{(s+5)}$$

$$Y(s) = \frac{3}{5} \cdot \frac{1}{s} + \frac{2}{5} \cdot \frac{1}{(s+5)} = \frac{1}{5}\left[\frac{3}{s} + \frac{2}{(s+5)}\right]$$

$$Y(s) = \frac{1}{5}\left[\frac{3}{s} + \frac{2}{(s+5)}\right]$$

$$y(t) = \frac{1}{5}\left[3 \cdot u(t) + 2 \cdot e^{-5t} u(t)\right]$$

$$y(t) = \frac{1}{5}\left[3u(t) + 2 \cdot e^{-5t} u(t)\right]$$

4. matrices $C = \begin{matrix} 1 & 1 \\ 1 & 2 \end{matrix}$

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Trace of matrices $= 1 + 2 = 3$

determinant of matrices $= \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$

$= 2 - 1 = 1$

$X + y = 3$

$Xy = 1$

$(X-y)^2 = (X+y)^2 - 4 \cdot y$

$(X-y)^2 = 9 - 4 \cdot 1 = 5$

$X - y = 2.236$

$X + y = 3$

$\underline{X - y = 2.236}$

$2X = 5.236$

$X = 2.618$

$y = 3 - X = 0.382$

5. $\frac{d^2 y(t)}{dt} + a\frac{dy(t)}{dt} + by(t) = x(t)$

$$\begin{cases} z_1 = z_2 \\ z_2 = bz_1 - az_2 + x(t) \end{cases}$$

$z = Az + Bx$

$A = \begin{bmatrix} 0 & 1 \\ b & -a \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$, $z \begin{bmatrix} z_1 \\ z_2 \end{bmatrix}$, and $x = \begin{bmatrix} x \end{bmatrix}$

$|ST - A| = \det \begin{bmatrix} s & -1 \\ -b & s+a \end{bmatrix} = s(s+a) - b = s^2 + as - b = 0$

$s_{1,2} = \frac{-a \pm \sqrt{a^2 + 4b}}{2} < 0$

The stable condition of this system is that : $a > 0$, $b < 0$

6. $\frac{Y(S)}{X(S)} = \frac{\frac{1}{CS}}{SL + R + \frac{1}{CS}}$ $(R = L = C = 1)$

$\frac{Y(S)}{X(S)} = \frac{1}{S^2 + S + 1}$ $S = -1 \pm \sqrt{3}i$ (system is stable)

$\frac{X(j\omega)}{X(j\omega)} = \frac{1}{(-4 + 2j + 1)} = \frac{1}{\sqrt{9+4}} = \frac{1}{\sqrt{13}}$

$\Theta = \tan^{-1}(-3/2) = \pi - \tan^{-1}(3/2) = 56.30$

$y(t) = x(t)(|H(j\omega)|)(\angle\Theta)$

$y(t) = \frac{10\cos(2t + 123.6)}{\sqrt{13}}$ stable system