Ahmed El Qaouaq

System and Signals:

Test 3

(1)

a) $10\, e^{-2(t-2)}\, u(t-2)$

$\Rightarrow$ Applying Laplace transform

$$\Rightarrow 10\, \mathcal{L}\left\{ e^{-2(t-2)}\, u(t-2) \right\}$$

$$\Rightarrow 10\, e^{-2s} \cdot \cancel{e^{-2s}} \cdot \mathcal{L}\left\{ e^{-2t}\, u(t) \right\}$$

$$\Rightarrow 10\, e^{-2s} \cdot \frac{1}{s+2} = \boxed{\frac{10 e^{-2s}}{s+2}}$$

b) $\sin(2(t-1))\, u(t-1)$

Applying Laplace transform:

$$\Rightarrow e^{-s} \cdot \frac{2}{s^2+4} = \boxed{\frac{2e^{-s}}{s^2+4}}$$

(2)

a) $X(s) = \dfrac{1}{s(s-2)}$

$$\frac{1}{s(s-2)} = \frac{A}{s} + \frac{B}{s-2}$$

$$\Rightarrow \frac{1}{s(s-2)} = \cancel{\frac{s-2+s}{s(s-2)}}\ \cancel{\frac{2s-2}{s(s-2)}} = \frac{As-2A+Bs}{s(s-2)}$$

$$\begin{cases} A+B=0 \\ -2A=1 \end{cases} \Rightarrow \begin{cases} A+B=0 \\ A=-\frac{1}{2} \end{cases} \Rightarrow \begin{cases} B=\frac{1}{2} \\ A=-\frac{1}{2} \end{cases}$$

$$X(s) = -\frac{1}{2s} + \frac{1}{2(s-2)}$$

$\Rightarrow$ ~~A~~Applying inverse laplace transform:

$$\boxed{x(t) = -\frac{1}{2} + \frac{1}{2} e^{2t}}$$

b) $$X(s) = \frac{2}{s^2+s+1}$$

$$\Rightarrow X(s) = \frac{2}{s^2 + 2 \cdot \frac{1}{2} \cdot s + \left(\frac{1}{2}\right)^2 - \left(\frac{1}{2}\right)^2 + 1}$$

$$\Rightarrow X(s) = \frac{2}{\cancel{s^2}\left(s+\frac{1}{2}\right)^2 - \frac{1}{4} + 1}$$

$$= \frac{2}{\left(s+\frac{1}{2}\right)^2 + \frac{3}{4}}$$

$\Rightarrow$ Applying Laplace transform

$$\Rightarrow 2 \cdot \left(\frac{2}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2} t\right)\right)$$

$$\boxed{x(t) = \frac{4}{\sqrt{3}} e^{-t/2} \sin\left(\frac{\sqrt{3}}{2} t\right)}$$

(3) $\frac{dy(t)}{dt} + 5y(t) = 3u(t) \quad y(0) = 1$

$y' + 5y = 3u(t), \quad y(0) = 1.$

Laplace transform

$(s\mathcal{L}(y) - 1) + 5\mathcal{L}(y) = \mathcal{L}(3u(t))$

$\Rightarrow s\mathcal{L}(y) - 1 + 5\mathcal{L}(y) = \frac{3}{s}$

$\Rightarrow s\mathcal{L}(y) + 5\mathcal{L}(y) = \frac{3}{s} + 1.$

$\Rightarrow \mathcal{L}(y)(s+5) = \frac{3+s}{s}$

$\Rightarrow \mathcal{L}(y) = \frac{\frac{3+s}{s}}{s+5}$

$\Rightarrow \mathcal{L}(y) = \frac{3+s}{s(s+5)}$

$\frac{3+s}{s(s+5)} = \frac{A}{s} + \frac{B}{s+5}$

$\Rightarrow \frac{3+s}{s(s+5)} = \frac{As + 5A + Bs}{s(s+5)} = \frac{s(A+B) + 5A}{s(s+5)}.$

$$\begin{cases} A+B=1 \\ 5A=3 \end{cases} \Rightarrow \begin{cases} A+B=1 \\ A=3/5 \end{cases} \Rightarrow \begin{cases} B=1-3/5 \\ A=3/5 \end{cases}$$

$$\Rightarrow \begin{cases} B=2/5 \\ A=3/5 \end{cases}$$

$$\Rightarrow \frac{3}{5s} + \frac{2}{5(s+5)} \Rightarrow$$

$$\Rightarrow \boxed{y(t) = \frac{3}{5} + \frac{2}{5} e^{5t}}$$

(4) $C = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

$$\det\left(\lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\right) = 0$$

$$\Rightarrow \det\left(\begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} - \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\right) = 0.$$

$$\Rightarrow \det\left(\begin{bmatrix} \lambda-1 & -1 \\ -1 & \lambda-2 \end{bmatrix}\right) =$$

$$(\lambda-1)(\lambda-3) - 1 = 0.$$

$$\Rightarrow \lambda^2 - 3\lambda - \lambda + 3 = 0$$

$\lambda^2 - 4\lambda + 3 = 0$

$\Delta = 16 - 4(1.3) = 16 - 12 = 4 \Rightarrow \sqrt{\Delta} = 2.$

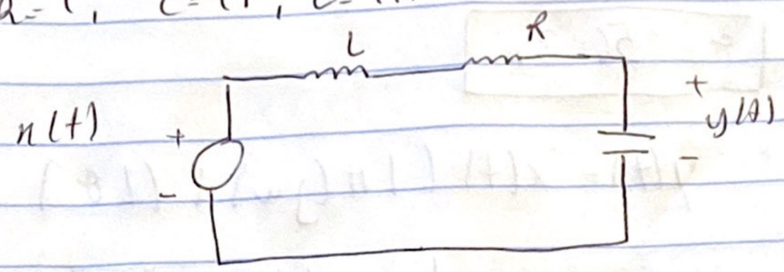
$\lambda_1 = \frac{4-2}{2} = \frac{2}{2} = 1$

$\lambda_2 = \frac{4+2}{2} = \frac{6}{2} = 3$

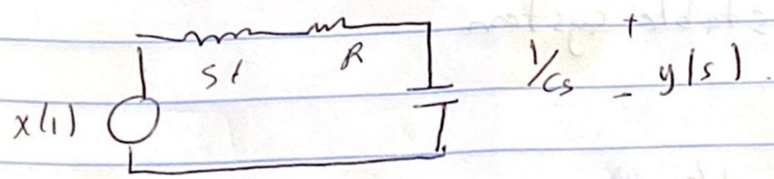
therefor the eigenvalue is $\boxed{\lambda = 1 \text{ and } \lambda = 3}$

⑥ Given

$R = 1, \quad C = 1F, \quad L = 1H.$



$n(t) = 10\cos(2t).$



$\frac{Y(s)}{X(s)} = \frac{\frac{1}{cs}}{sL + R + 1/cs}$ $(R = L = C = 1).$

$\frac{Y(s)}{X(s)} = \frac{1}{s^2 + s + 1}$

roots of equation $s = -1 \pm \sqrt{3}\, i$

system stable

$$\frac{Y(j\omega)}{X(j\omega)} = \frac{1}{-\omega^2 + j\omega + 1} \qquad x(t) = 10\cos(2t)$$

$$\omega = 2$$

$$H(j\omega) = \left(\frac{Y(j\omega)}{X(j\omega)}\right) = \left|\frac{1}{-4+2j+1}\right| = \frac{1}{\sqrt{9+4}}$$

$$= \frac{1}{\sqrt{13}}$$

$$\theta = \tan^{-1}\left(\frac{-3}{2}\right) = \pi - \tan^{-1}(3/2)$$

$$\boxed{\theta = 56.30}$$

$$y(t) = x(t)\,(|H(j\omega)|)\,(\angle\theta)$$

$$y(t) = \frac{10\cos(2t + 23.6)}{\sqrt{13}}$$

stable system

(5) $\frac{d^2 y(t)}{dt^2} + a\frac{dy}{dt} + b\,y(t) = x(t).$

$\cancel{\frac{d}{dt}}$ ~~E(t)~~ =

(5)

$\dot{y} = \frac{dy}{dt}$

so $\ddot{y} + a\dot{y} + by = x(t)$.

$x_1 = y$ also $\dot{x}_1 = x_2$

$x_2 = \dot{y}$ $\dot{x}_2 = x_3$.

$x_3 = \ddot{y}$

$\dot{x}_2 + a\dot{x}_1 + bx = x(t)$.

(a) $\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -b & -a \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} x(t)$

$\therefore \dot{x}_2 = -ax_2 - bx_1 + x(t)$.

$\therefore y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$