System and Signal

Homework 5 p.265

EOCP 5.1

1.) $2u(t-10)+10\delta(t-1)$

- $u(t) \xleftrightarrow[\text{transform}]{\text{Laplace}} \frac{1}{s}$ and $\delta(t) \xleftrightarrow[\text{transform}]{\text{Laplace}} 1$

★ If $x(t) \leftrightarrow X(s)$ then $x(t-t_0) \leftrightarrow e^{-st_0} X(s)$

- $u(t-t_0) \xleftrightarrow{L.T.} e^{-10s} \cdot \frac{1}{s}$ and $\delta(t-1) \leftrightarrow e^{-s} \cdot 1$

- $\boxed{2u(t-10)+10\delta(t-1) \leftrightarrow \frac{2e^{-10s}}{s} + e^{-s}}$

3. $\cos(3(t-5))u(t-5)$ delay $= 5$; $t_0 = 5$

- Using the formula from Equation (1.)

$(\cos w_0 t \cdot u(t)) \xleftrightarrow{F.T.} \pi[\delta(w-w_0)+\delta(w+w_0)]$

$\pi[\delta(w-3)+\delta(w+3)] \cdot e^{-jw5}$

5. $(t-1)e^{-a(t-1)}u(t-1)$

$u(t) \leftrightarrow 1/s$

- If $x(t) \leftrightarrow X(s)$ then $t \cdot (t) \leftrightarrow \frac{-dX(s)}{ds}$

$tu(t) \leftrightarrow \frac{-d}{ds}(X_s) \rightarrow tu(t) \leftrightarrow 1/s^2$

If $x(t) \leftrightarrow X(s)$ then $e^{-at}x(s) \leftrightarrow X(s+a)$

$e^{-at}, tu(t) \leftrightarrow 1/(s+a)^2$

$= \boxed{(t-1)e^{-a(t-1)}u(t-1) \leftrightarrow \frac{e^{-s}}{(s+a)^2}}$

EOCP 5.2

3. $(e^{-3t} + \sin(t) u(t))$

$f(t) = (e^{-3t} + \sin t) u(t)$

$\mathcal{L}(a) + \mathcal{L}(b) = \mathcal{L}(a+b)$

$L(f(t)) = F(s) = L(e^{-3t} u(t))$

$= \frac{1}{s+3} + \frac{1}{s^2+1}$

$= \frac{s^2+s+4}{(s+3)(s^2+1)}$

$= \frac{s^2+s+4}{s^3+3s^2+s+8}$

$$\boxed{L[(e^{-3t}+\sin t)u(t)] = \frac{s^2+s+4}{s^3+3s^2+s+3}}$$

EOCP 5.3

3.) $[e^{-t}u(t) + e^{-2t}u(t)] \cdot \sin(t)u(t)]$

$L[(e^{-t}u(t) + e^{-2t}u(t)) \cdot \sin t\, u(t)]$

$= \left[\frac{1}{[s+1][s^2+1]} + \frac{1}{[s+2][s^2+1]}\right]$

$$= \boxed{\frac{2s+3}{(s^2+1)(s^2+3s+2)}}$$

5.) $\delta(t) \cdot \sin(10t)u(t) - \delta(t-1) \cdot u(t)$

$L[\delta(t) \cdot (\sin 10t\, u(t) - \delta(t-1) \cdot u(t)]$

$= 1 \cdot \left[\frac{10}{s^2+10^2} - e^{-1}\right] \cdot \frac{1}{s}$

$$\boxed{\frac{10}{s[s^2+100]} - \frac{e^{-t}}{s}}$$

EOCP 5.4

2. $\frac{d^2}{dt} y + 3\frac{dy}{dt} + 2y = x(t)$

$y(s) = [s^2+3s+2] - 2s - 6 = 1/s$

$[s^2+3s+2] - 2s - 6] = \frac{1}{s+(2s+6)}$

$= \frac{1}{s}(s^2+3s+2) + \frac{(2s+6)}{s^2+3s+2}$

$= \frac{(1+2s^2+6s)}{s(s+1)(s+2)}$

$= \boxed{\left[\left(\frac{1}{2s}\right) + \left(\frac{3}{(s+1)}\right) - \left(\frac{2}{(s+2)}\right)\right]}$

4. $\frac{d^2y}{dt^2} + 5y = x(t) = e^{t}u(t),\ y(0) = 0,\ \frac{d}{dt}y(0) = -3$

$= \frac{d^2y}{dt^2} + 5y = e^{-t}u(t)$

$= \frac{dy}{dt} - \sqrt{5}\, A \sin\sqrt{5}\, t u(t) + \sqrt{5}\, B \cos\sqrt{5}\, t u(t) - \frac{1}{6} e^{t} u(t)$

$= \sqrt{5}\, B2 - 3 + \frac{1}{6}$

$= \frac{-3 \cdot 6 + 1}{6}$

$= \frac{-16+1}{6}$

$= \boxed{\frac{-17}{6}}$

$y(t) = \frac{-1}{6}\cos\sqrt{5}\, t u(t) - \frac{17}{6\sqrt{5}} \sin\sqrt{5}\, t u(t) + \frac{1}{6} e^{-t} u(t)$

$\boxed{y(t) = \frac{-1}{6}\cos\sqrt{5} + u(t) - \frac{17}{6\sqrt{5}} \sin\sqrt{5}\, t u(t) + \frac{1}{6} e^{-t} u(t)}$

EOCP 5.5

6.) $\frac{e^{-s}s+s}{s^2+4s+4}$

$f(s)=\frac{se^{-s}}{(s+2)^2}+\frac{s}{(s+2)^2}$

$LT^{-1}\left[\frac{1}{(s+2)^2}\right]$

$=te^{-2t}$

$F(t)=\frac{d}{dt}\left[(t-1)e^{-2t+2}\right]+te^{-2t}$

$=\boxed{3e^{-2t+2}-2te^{-2t+2}+e^{-2t}}$

8. $\frac{s^2}{s^2+7s+10} \rightarrow \frac{s^2}{s^2+7s+10}=\frac{A}{(s+2)}+\frac{B}{(s+5)}$

$=\frac{s^2}{(s^2+7s+10)}$

$=\frac{A(s+5)+B(s+2)}{(s+5)\ (s+2)}$

$y(s)=\frac{3}{4}\cdot(s+2)-\frac{3}{2s}\cdot(s+5)$

$\boxed{y(t)=\frac{[3\cdot(e^{-2t})]}{4}-\frac{[3\cdot(e^{-5t})]}{2s}}$

EOCP 5.6

Figure 5.27 $u(t) \rightarrow \boxed{te^{-2t}u(t)} \rightarrow \boxed{e^{-2t}u(t)} \rightarrow y(t)$

$$u(t) \cdot h_1(t) = \int_{-\infty}^{\infty} h_1(t)\, u(t-h)$$

$$y_1(t) = \int_0^t te^{-2t}\, dt$$

$$= t\int_0^t e^{-2t}\, dt - \int_0^t 1 \int e^{-2t}\, dt$$

$$= \left[\frac{Te^{-2t}}{2}\right]_0^t - \left[\frac{e^{-2t}}{4}\right]_0^t$$

$$= \left[\frac{-t}{2}e^{-2t} + \frac{1-e^{-2t}}{4}\right]$$

$$y_1(t) = \frac{-te^{-2t}}{2}u(t) + \frac{1}{4}u(t) \frac{-e^{-2t}}{4}(u)t$$

$$y(t) = e^{-2t}\int_0^t \cdot \left(\frac{-T}{2}e^{-2t}e^{2t} + \frac{1}{4}e^{2t} - \frac{e^{-2t} \cdot e^{2t}}{4}\right) dt$$

$$y(t) = e^{-2t}\int_0^t \left(\frac{-t}{2} + \frac{1}{4}e^{2t} - \frac{1}{4}\right) dt$$

$$y(t) = e^{-2t} - \int_0^t \frac{t}{2}\, dt + \frac{1}{4}\int_0^t e^2\, dt - \frac{1}{4}\int_0^t dt$$

$$\boxed{y(t) = \frac{-t^2}{4}e^{-2t} + \frac{1}{8} - \frac{e^{-2t}}{8}u(t) - \frac{te^{-2t}}{4}u(t) - \frac{t^2}{4}e^{-2}u(t)}$$