1). $2+1=3$ and $4+5=8$ then $2+2=4$ or $2+2=5$

$2+1=3$ T. $\quad$ $2+2=4$ True

$4+5=8$ F $\quad$ $2+2=5$ False

$\therefore$ If T and F the T or F

T and F = F $\quad$ T or F = T

$\therefore$ If F then T $\quad F \Rightarrow T$ alway True

So the Truth value is "T" True.

2). $Q(x): 2x+1=2|x|+1 \quad \forall x Q(x)$

$-Q(x) = 2x+1 = 2(x)+1$: True For all $x \geq 0$

$-Q(x) = 2x+1 = -2(x)+1$: False for $x<0$

$\therefore Q(x) = 2x+1 = 2|x|+1$ is False "F" since it is not True for $x<0$, Truth value is F

3.

Let's consider

$A=\{1,2,3,6\} \quad B=\{1,6\} \quad C=\{1,2,3,4,5\}$

$A \cup C = \{1,2,3,4,5,6\}$

And $B \cup C = \{1,2,3,4,5,6\}$

$\therefore A \cup C = B \cup C$ but $A \neq B$

Truth value is "False"

4). $(\neg p \wedge (p \vee q)) \Rightarrow q$

p	q	$\neg p$		$p \vee q$	$\neg p \wedge (p \vee q)$	$\neg p \wedge (p \vee q)) \Rightarrow q$
T	T	F		T	F	T
T	F	F		T	F	T
F	T	T		T	T	T
F	F	T		F	F	T

All trues

$\therefore (\neg p \wedge (p \vee q)) \Rightarrow q$ Tautology

5. $g(x) = \lceil x+1 \rceil$ and $S = \{-1, 0, 1\}$

$g^{-1}(-1) = \{ x \in \mathbb{R} \mid g(x) = -1$

$-2 < x+1 \leq -1 \Rightarrow -3 < x \leq -2$

$g^{-1}(-1) = (-3, -2)$

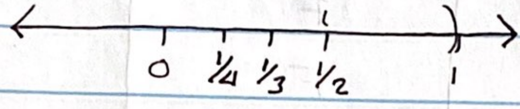
$g^{-1}(0) = \{x \in \mathbb{R} \mid g(x) = 0\}$, $-1 < x+1 \leq 0 \Rightarrow -2 < x \leq -1$

$g^{-1}(0) = -2, -1]$

$g^{-1}(1) = [-1, 0]$

$g^{-1}(S) = (-3, -2] \cup (-2, -1] \cup (-1, 0) = \boxed{[-3, 0]}$

6). A) Find $\bigcup_{n=1}^{\infty} A_n = \bigcup_{n=1}^{\infty} (1/n, 1) = (1,1) \cup (1/2, 1) \cup (1/3, 1) \cup \ldots$



$\therefore \bigcup_{n=1}^{\infty} A_n = \bigcup_{n=1}^{\infty} \left(\frac{1}{n}, 1\right) = (0,1)$

b. Find $\bigcap_{n=1}^{\infty} A_n = \bigcap_{n=1}^{\infty} (1/n, 1)$

$= (1,1) \cap (1/2, 1) \cap (1/3, 1) \cap \ldots$

As, Smallest interval is $(1,1) = \emptyset$ (Empty set)

$\therefore \bigcap_{n=1}^{\infty} A_n = \bigcap_{n=1}^{\infty} \left(\frac{1}{n}, 1\right) = \emptyset$

7). $f: Z \times Z \to Z$ Defined by $f(m,n) = 2m - n$

F is not one to one

Example: $\exists\ (5,5) + (10,15) \in Z \times Z$

$f(5,5) = f(10,15) \quad 5 \neq 10,\ 5 \neq 15$

$f(5,5) = 5 \times 2 - 5 \qquad f(10,15) = 2 \times 10 - 15$

$= 10 - 5 = 5 \qquad = 20 - 15$

$\therefore f(5,5) = f(10,5)$ but $(5,5) \neq (10,15) \qquad = 5$

- F not one to one

3) $f(x,x) = 2x - x$

$= x$

For Every $x \in Z\ \exists\ (x,x) \in Z \times Z$

Such that

$f(x,x) = x$

$\therefore$ f is onto function.

8). $g = \{(a,d),(b,b),(c,b),(d,c)\}$

$f = \{(a,b),(b,d),(c,a),(d,b)\}$

$f \circ g(a) = f(g(a)) = f(d) = b$
$f \circ g(b) = f(g(b)) = f(b) = d$
$f \circ g(c) = f(g(c)) = f(b) = d$
$f \circ g(d) = f(g(d)) = f(c) = a$
$f \circ g = \{(a,b),(b,d),(c,d),(d,a)\}$

9) P: "The file system is locked"

q: "New message will be queued."

R: "The system is functioning normally"

s: "The new messages will be sent to the message buffer.

1. $\neg P \rightarrow q$
2. $\neg P \leftrightarrow r$
3. $\neg q \rightarrow s$
4. $\neg P \rightarrow s$
5. $\neg s$

10). $p \wedge q \equiv \neg(p \Rightarrow \neg q)$

$LHS = p \wedge q$

$RHS = \neg(p \Rightarrow \neg q)$

$\neg(p \Rightarrow \neg q)$

$\equiv \neg(\neg p \vee \neg q)$ Logical equivivalence $p \Rightarrow q) \equiv (\neg p \vee q)$

De Morgan's law $\equiv \neg(\neg(p \wedge q))$

$\equiv (p \wedge q)$ Double negation law

thus $\neg(p \Rightarrow \neg q) \equiv p \wedge q$

$\boxed{\therefore p \wedge q \equiv \neg(p \Rightarrow \neg q)}$

11. $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee \neg r \vee \neg s) \wedge (\neg p \vee \neg q \vee \neg s)$
$(p \vee q \vee \neg s)$

$\Rightarrow (p \vee q \vee \sim r) \wedge (p \vee q \vee \sim s) \wedge (p \vee \sim r \vee \sim s) \wedge (\sim p \vee \sim q \vee \sim s)$

$\therefore A \wedge A = A$

$\Rightarrow (p \vee q) \wedge (\sim r \vee \sim s) \wedge (p \vee \sim r \vee \sim s) \wedge (\sim p \vee \sim q \vee \sim s)$
$\Rightarrow (p \vee q) \wedge (\sim r \vee \sim s) \wedge (1 \vee 0) \wedge (\sim p \vee \sim q \vee \sim s)$
$\Rightarrow (p \vee q) \wedge \sim(r \wedge s) \wedge \sim(p \wedge q \wedge s)$

Table

p	q	r	s	$p \vee q$	$r \wedge s$	$\sim(r \wedge s)$	$p \wedge q \wedge s$	$\sim(p \wedge q \wedge s)$	$(p \vee q) \wedge (\sim(r \wedge s)) \wedge \sim(p \wedge q \wedge s)$
F	F	F	F	F	F	T	F	T	F
F	F	F	T	F	F	T	F	T	F
F	F	T	F	F	F	T	F	T	F
F	F	T	T	F	T	F	F	T	F
F	T	F	F	T	F	T	F	T	T
F	T	F	T	T	F	T	F	T	T
F	T	T	F	T	F	T	F	T	T
F	T	T	T	T	T	F	F	T	F
T	F	F	F	T	F	T	F	T	T
T	F	F	T	T	F	T	F	T	T
T	F	T	F	T	F	T	F	T	T
T	F	T	T	T	T	F	F	T	F
T	T	F	F	T	F	T	F	T	T
T	T	F	T	T	F	T	T	F	T
T	T	T	F	T	F	T	F	T	T
T	T	T	T	T	T	F	T	F	F

$\therefore$ SATISFIABLE BASED ON TABLE