Ashleigh Reeves EE 330

Test 2

1. $\frac{1-2j}{-j-1} + j = \frac{(1-2j)(j+1)}{(-j-1)(-j+1)} + 1$

$= \frac{j - 2j^2 + 1 - 2j}{j^2 - j + j - 1} + 1$

$= \frac{-2j^2 - j + 1}{j^2 - 1}$

$= \frac{2 - j + 1}{-2} + 1$

$= \frac{3 - j}{-2} + 1$

$= -\frac{1}{2} + \frac{1}{2}j$

$M = \sqrt{(\frac{1}{2})^2 + (\frac{1}{2})^2}$

$= \sqrt{1/2}$

$\tan^{-1}(1) = 45°$

2. $3\frac{dy(t)}{dt} + 3y(t) = x(t) + 3\frac{dx(t)}{dt}$

$x(t) = e^{j\omega t}$

$y(t) = H(\omega)e^{j\omega t}$

$\frac{d}{dt}y(t) = j\omega H(\omega)e^{j\omega t}$, $\frac{d}{dt}x(t) = j\omega e^{j\omega t}$

$j\omega H(\omega)\cancel{e^{j\omega t}} + 3(H(\omega)\cancel{e^{j\omega t}}) = \cancel{e^{j\omega t}} + 3(j\omega \cancel{e^{j\omega t}})$

$j\omega H(\omega) + 3H(\omega) = 1 + 3j\omega$

$H(\omega)(j\omega + 3) = 1 + 3j\omega$

$\boxed{H(\omega) = \frac{1 + 3j\omega}{j\omega + 3}}$

3. $[e^{-3t}u(t) * u(t)] + u(t) * u(t)$

$\mathcal{F}(e^{-3t}u(t)) = \frac{1}{a+j\omega} = \frac{1}{3+j\omega}$

$\mathcal{F}u(t) = \frac{1}{j\omega}$

$X(j\omega) = \left(\frac{1}{3+j\omega}\right)\left(\frac{1}{j\omega}\right) + \left(\frac{1}{j\omega}\right)\left(\frac{1}{j\omega}\right)$

$= \frac{j\omega + (3+j\omega)}{(j\omega)^2(3+j\omega)}$

$$\boxed{X(j\omega) = \frac{3+2j\omega}{(j\omega)^2(3+j\omega)}}$$

4. $X(j\omega) = \frac{1}{(-\omega^2 + 5j\omega + 6)} = \frac{1}{(j\omega+2)(j\omega+3)}$

$j\omega$

$= \frac{1}{(j\omega+2)} - \frac{1}{(j\omega+3)}$

so,

$x(t) = (e^{-2t} - e^{-3t})u(t)$

5. $e^{-a(t-b)}u(t-b) + u(t-2)$

$\mathcal{F}(j\omega) = \frac{1}{(a+j\omega)}e^{-j\omega b} + \frac{1}{j\omega}e^{-2j\omega}$

6. $T=4$

$x(t) \begin{cases} 2+t, & 0 \le t \le 4 \\ 0, & 0 \end{cases}$

$a_0 = \frac{1}{4}\int_0^4 x(t)\,dt$

$= \frac{1}{4}\int_0^4 (2+t)\,dt$

$= \frac{1}{4}\left[2t + \frac{t^2}{2}\right]_0^4$

$= \frac{1}{4}[8+8]$

$= 4$

$a_n = \frac{1}{2}\int_0^4 (2+t)\cos(nt)\,dt$

$= \frac{1}{2}\int_0^4 [2\cos(nt) + t\cos(nt)]\,dt$

$= \frac{1}{2}(2\sin(nt) + \frac{1}{2}(\frac{t}{n}\sin(nt) + \frac{1}{n^2}\cos(nt)$

$a_n = \sin(4n) + \frac{1}{2}(\frac{t}{n}\sin(4n) + \frac{1}{n^2}\cos(4n)$