a) $\lim\limits_{x \to 0} \dfrac{x}{\sin \frac{x}{2}}$

$= \lim\limits_{x \to 0} \dfrac{2 \cdot x}{2 \cdot \sin x/2}$

$= 2 \lim\limits_{x \to 0} \dfrac{x}{\sin x}$

$2 \cdot 1 = 2$

b. Since the denominator is a fraction, I am able to multiply the denominator & the numerator by 2. I carried the 2 from the numerator, and then cancelled out the denominator, which led to $x/\sin x$ ~~equalling~~ equalling 1. 2 times 1 became the limit

3. a) $\lim\limits_{x \to 0} \dfrac{\sin ax}{\sin bx}$

$\dfrac{a}{b} \lim\limits_{x \to 0} \dfrac{\sin x}{\sin x} = \dfrac{a}{b}$

b. $\lim_{x \to 0} \frac{\tan x - \sin x}{x \cos x}$

$= \lim_{x \to 0} \frac{\left(\frac{\sin x}{\cos x} - \sin x\right) \cos x}{(x \cos x) \cos x}$

$= \lim_{x \to 0} \frac{\sin x - \sin x \cos x}{x \cos^2 x}$

$= \lim_{x \to 0} \frac{\sin x (1 - \cos x)}{x \cos^2 x}$

$= \lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1 - \cos x}{\cos^2 x}$

$\lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{1 - \cos x}{\cos^2 x}$

$1 \cdot \frac{1 - \cos(0)}{\cos^2 0} = 1 \cdot \frac{1-1}{1} = 0$

c. $\lim_{x \to 0} \frac{1 - \cos x}{x} = 2 \sin^2\left(\frac{x}{2}\right)$ so

$= \frac{1 - \cos x}{x} = \left(\frac{x}{4}\right)\left(\frac{\sin \frac{x}{2}}{\frac{x}{2}}\right)^2$

$= \lim_{x \to 0} \frac{1 - \cos x}{x} = \lim_{x \to 0} \left(\frac{x}{4}\right)\left(\frac{\sin\left(\frac{x}{2}\right)}{x/2}\right)^2$

$= 0 \cdot 1 = 0$

d. $\lim\limits_{x \to 0} \dfrac{\sin x}{\tan x} = \lim\limits_{x \to 0} \dfrac{\sin x}{\frac{\sin x}{\cos x}}$

$= \lim\limits_{x \to 0} \dfrac{\cancel{\sin x}}{1} \cdot \dfrac{\cos x}{\cancel{\sin x}}$

$= \lim\limits_{x \to 0} \dfrac{\cos x}{1} = 1$

e. $\lim\limits_{x \to 0} \dfrac{\sin^2 x \cos x}{1 - \cos x}$ $\quad 1 - \cos^2 x$

$= \lim\limits_{x \to 0} \dfrac{\sin x}{x} \cdot \dfrac{\sin x}{1} \cdot \dfrac{\cos x}{1 - \cos x}$

$= \lim\limits_{x \to 0} \dfrac{(1 - \cos^2 x)\cos x}{1 - \cos x} \cdot \dfrac{(\cancel{1 - \cos x})(1 + \cos x)\cos x}{\cancel{(1 - \cos x)}}$

$= \lim\limits_{x \to 0} (1 + \cos x)\cos x = \cos(0) + \cos(0)$

$= 1 + 1$

$= 2$

f - $\lim\limits_{x \to 0} \dfrac{1 - \cos x}{\tan x}$

$= \lim\limits_{x \to 0} \dfrac{2\sin^{x} x}{1} \cdot \cancel{Si} \dfrac{\cancel{\sin x}}{\cos x}$

$\lim\limits_{x \to 0} \dfrac{2\sin(0)}{\cos(0)} = 0$

4a. $\lim_{x \to 0} \frac{\sin x}{x + \sin x}$

$= \lim_{x \to 0} \frac{\sin x}{x \cancel{+ \sin x}}$

$= \lim_{x \to 0} \; 1 + \frac{\sin x}{x}$

$= \lim_{x \to 0} \frac{\frac{\sin x}{x}}{1 + \frac{\sin x}{x}} = \frac{1}{2}$

b. I evaluated this as an equation that can be written as a ~~f~~fraction because both the numerator & denominator have sin x in them.