Denzel Paul
Homework 3

Question 1

g. $\lim_{x \to 0} \frac{\sin^3 2x}{\sin^3 3x}$

$\lim_{x \to 0} \left(\frac{\sin 2x}{3 \sin 3x}\right)^3$

$\frac{2^3}{3^3} \lim_{x \to 0} \frac{\sin 2x}{\sin 3x}$

$= \frac{8}{27}$

h.) $\lim_{x \to 0} \frac{\sin x}{7x}$

$\lim_{x \to 0} \frac{7 \cdot \sin x}{7 \cdot 7x}$

$\frac{1}{7} \lim_{x \to 0} \frac{\sin 7x}{7x}$

$\frac{1}{7} \cdot 1 = \frac{1}{7}$

i. $\lim_{x\to 0} \frac{\sin 5x}{\sin 6x}$

$\lim_{x\to 0} \frac{5 \cdot \frac{\sin 5x}{5x}}{6 \cdot \frac{\sin 6x}{6x}}$

$\frac{5}{6} \lim_{x\to 0} \frac{\frac{\sin 5x}{5x}}{\frac{\sin 6x}{6x}}$

$\frac{5}{6} \cdot 1 = \frac{5}{6}$

j.) $\lim_{x\to 0} \frac{\sin 6x}{6}$

$\lim_{x\to 0} \frac{\sin 6(0)}{6}$

$= 0$

K. $\lim_{x\to 0} \frac{\sin^2 3x}{x^2}$

$\lim_{x\to 0} \frac{\sin 3x}{x} \cdot \frac{\sin 3x}{x}$

$\lim_{x\to 0} \frac{3 \cdot \sin 3x}{3 \cdot x} \cdot \frac{3 \sin 3x}{3 \cdot x}$

$(3)(3) \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \frac{\sin 3x}{3x}$

$(3)(3)(1)(1) = 9$

L.) $\lim_{x\to 0} \frac{\tan x}{4x}$

$\lim_{x\to 0} \frac{\frac{\sin x}{\cos x}}{4x}$

$\lim_{x\to 0} \frac{4 \cdot \sin x}{4 \cdot x \cos 4x}$

$\frac{1}{4} \lim_{x\to 0} \frac{\sin x}{4x} \cdot \frac{1}{\cos 4x}$

$\frac{1}{4} \cdot 1 \cdot 1 = \frac{1}{4}$

Question 2

a. $\lim\limits_{x \to 0} \dfrac{x}{\sin \frac{x}{2}}$

$\lim\limits_{x \to 0} \dfrac{2 \cdot x}{2 \cdot \sin \frac{x}{2}}$

$2 \lim\limits_{x \to 0} \dfrac{x}{\sin x}$

$2 \cdot 1 = 2$

b. The denominator has a fraction of $\frac{x}{2}$ which means I'm able to multiply both the numerator and denominator by 2. After the 2 was carried of from the numerator, the denominator cancelled out leaving with ~~$\sin x$~~ $\sin x$ as $\frac{x}{\sin x} = 1$. Therefor $2 \times 1 = 2$ as the limit.

3a. $\lim\limits_{x \to 0} \dfrac{\sin ax}{\sin bx}$

$\lim\limits_{x \to 0} \dfrac{\sin ax}{\sin bx}$

$\dfrac{a}{b} \lim\limits_{x \to 0} \dfrac{\sin x}{\sin x} = \dfrac{a}{b}$

b.) $\lim\limits_{x \to 0} \dfrac{\tan x - \sin x}{x \cos x}$

$\lim\limits_{x \to 0} \dfrac{\left(\frac{\sin x}{\cos x} - \sin x\right) \cancel{\cos x}}{(x \cos x) \cos x}$

$\lim\limits_{x \to 0} \dfrac{\sin x - \sin x \cos x}{x \cos^2 x}$

Continue $\longrightarrow$

$$\lim_{x \to 0} \frac{\sin x(1-\cos x)}{x \cos^2 x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{1-\cos x}{\cos^2 x}$$

$$\lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{1-\cos x}{\cos^2 x}$$

$$1 \cdot \frac{1-\cos(0)}{\cos^2 0} = 1 \cdot \frac{1-1}{1} = 0$$

c. $$\lim_{x \to 0} \frac{1-\cos x}{x}$$

$$\lim_{x \to 0} \frac{1-\cos x}{x} \cdot \frac{1+\cos x}{1+\cos x}$$

$$\lim_{x \to 0} \frac{1-\cos^2 x}{x(1+\cos x)}$$

$$\lim_{x \to 0} \frac{\sin^2(x)}{x(1+\cos(x))}$$

$$\lim_{x \to 0} \frac{\sin x}{x} \lim_{x \to 0} \frac{\sin x}{1+\cos(x)} = 0$$

$$1 \cdot 0$$

d. $\lim_{x \to 0} \frac{\sin x}{\tan x}$ $\lim_{x \to 0} \frac{\sin x}{\frac{\sin x}{\cos x}}$

$\lim_{x \to 0} \frac{\cancel{\sin x}}{1} \cdot \frac{\cos x}{\cancel{\sin x}}$

$\lim_{x \to 0} \frac{\cos x}{1} = 1$

e. $\lim_{x \to 0} \frac{\sin^2 x \cos x}{1 - \cos x}$ $\rightarrow 1 - \cos^2 x$

$\lim_{x \to 0} \frac{\sin x}{x} \cdot \frac{\sin x}{1} \cdot \frac{\cos x}{1 - \cos x}$

$\lim_{x \to 0} \frac{(1 - \cos^2 x) \cos x}{1 - \cos x} = \frac{\cancel{(1 - \cos x)}(1 + \cos x) \cos x}{\cancel{(1 - \cos x)}}$

$\lim_{x \to 0} (1 + \cos x) \cos x = \cos(0) + \cos(0)$

$= 1 + 1$

$= 2$

f. $\lim\limits_{x \to 0} \dfrac{1-\cos x}{\tan x} \qquad \dfrac{1-\cos x}{\frac{\sin x}{\cos x}}$

$\lim\limits_{x \to 0} \dfrac{2\sin^2 x}{1} \cdot \dfrac{\cancel{\sin x}}{\cos x}$

$\lim \dfrac{2\sin(0)}{\cos(0)} = 0$

4a. $\lim\limits_{x \to 0} \dfrac{\sin x}{x + \sin x} \qquad \dfrac{\frac{\sin x}{x}}{1+\frac{\sin x}{x}}$

$\lim\limits_{x \to 0} \dfrac{\sin x}{x} \qquad \lim\limits_{x \to 0} 1 + \dfrac{\sin x}{x}$

$\lim\limits_{x \to 0} \dfrac{\frac{\sin x}{x}}{1+\frac{\sin x}{x}} = \dfrac{1}{2}$

b. The ~~nu~~ numerator and denominator has $\sin x$ in it which can be written as a fraction of x.