

W3 - 1.6 Instantaneous Rates of Change

MHF4U

Jensen

1) Consider the graph shown.

a) State the coordinates of the tangent point

$$(5, 3)$$

b) State the coordinates of another point on the tangent line

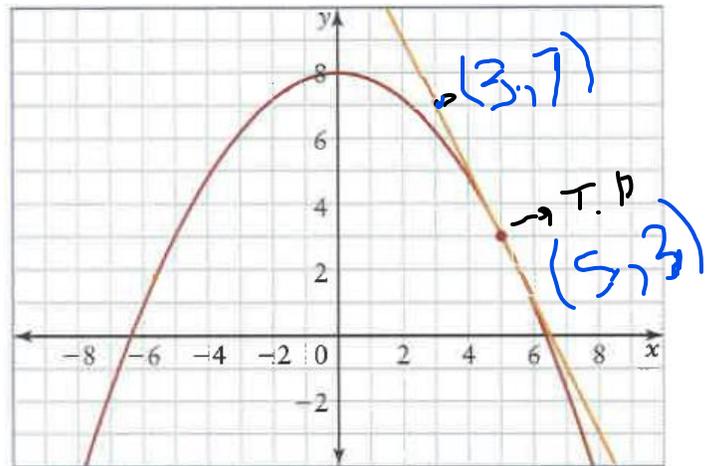
$$(3, 7)$$

c) Use the points you found to find the slope of the tangent line

$$m = \frac{7 - 3}{3 - 5} = \frac{4}{-2} = -2$$

d) What does the slope of the tangent line represent?

Instantaneous rate of change



2)a) At each of the indicated points on the graph, is the instantaneous rate of change positive, negative, or zero?

$$A = +ve \quad B = 0 \quad C = -ve$$

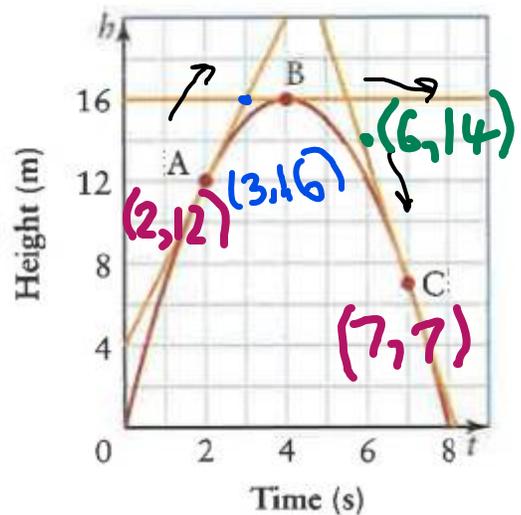
b) Estimate the instantaneous rate of change at points A and C.

$$\frac{dy}{dx} A = \frac{16 - 12}{3 - 2} = 4$$

$$\frac{dy}{dx} C = \frac{14 - 7}{6 - 7} = -7$$

$$\therefore \text{IROC @ A} = 4 \text{ and } \text{IROC @ C} = -7$$

Height of a Tennis Ball



c) Interpret the values in part b) for the situation represented by the graph.

A = 4 means function is increasing at that point and IROC is +ve

C = -7 means function is decreasing at that point and IROC is -ve

3) Use the graph of each function to estimate the instantaneous rate of change at $x = 2$ by drawing a tangent line and calculating its slope.

@ $x = 2 ; y = 3$

a) $3x^2 - 5x + 1$

$$\frac{\Delta y}{\Delta x} = \frac{f(2+h) - f(2)}{h} =$$

$$\frac{[3(h+2)^2 - 5(h+2) + 1] - 3}{h}$$

$$\frac{3h^2 + 12h + 12 - 5h - 10 + 1 - 3}{h}$$

$$= \frac{3h^2 + 7h}{h} = \cancel{h}(3h + 7) = 3h + 7$$

$$\therefore \text{slope} = 3(0) + 7 = 7$$

b) $\sqrt{x+2}$

@ $x = 2 ; y = 2$

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4 - 2}{10 - 2}$$

$$= \frac{2}{8}$$

$$= \frac{1}{4}$$

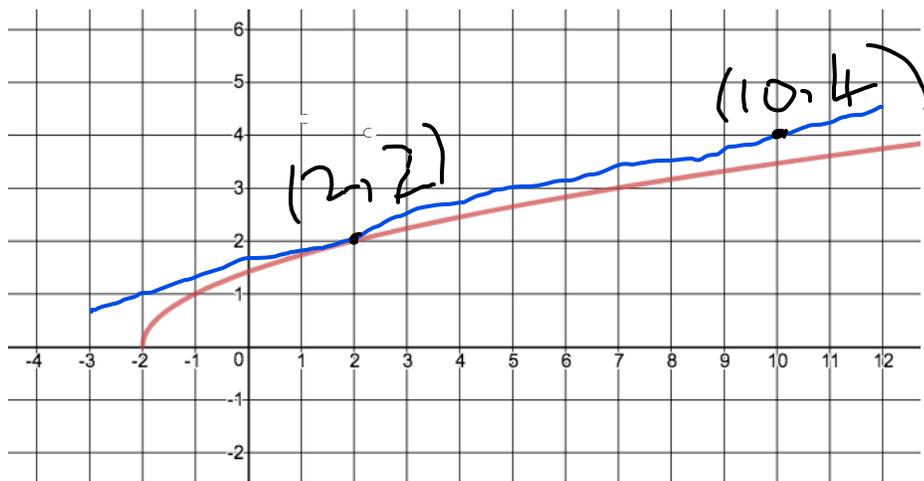
\therefore slope of tangent is $\frac{1}{4}$



(3, 10) OR

$$\frac{\Delta y}{\Delta x} = \frac{10 - 3}{3 - 2} = 7$$

\therefore slope of tangent is 7.



4) Verify your answers from question #3 by calculating the LIMIT of the secant slopes as you approach $x = 2$.

a)

Interval	Δy	Δx	Slope of secant = $\frac{\Delta y}{\Delta x}$
$2 \leq x \leq 2.5$			
$2 \leq x \leq 2.1$			
$2 \leq x \leq 2.01$			
$2 \leq x \leq 2.001$			

b)

Interval	Δy	Δx	Slope of secant = $\frac{\Delta y}{\Delta x}$
$2 \leq x \leq 2.5$			
$2 \leq x \leq 2.1$			
$2 \leq x \leq 2.01$			
$2 \leq x \leq 2.001$			