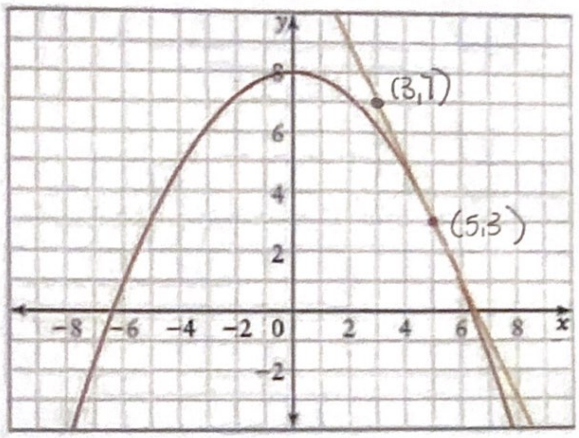
W3 – 1.6 Instantaneous Rates of Change
MHF4U
Jensen Terrencia Miller

1) Consider the graph shown.



a) State the coordinates of the tangent point

(5,3)

b) State the coordinates of another point on the tangent line

(3,7)

c) Use the points you found to find the slope of the tangent line

m = -2

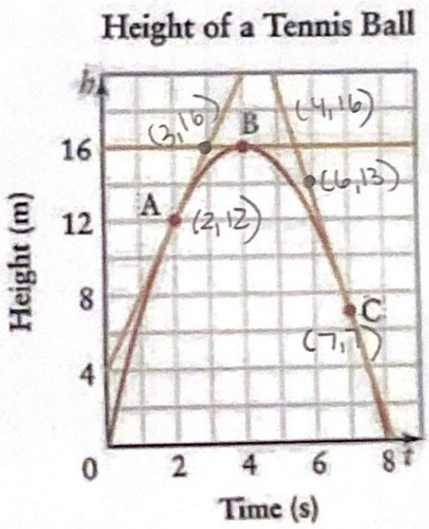
$m = \frac{3-7}{5-3} = \frac{-4}{2} = -2$

d) What does the slope of the tangent line represent?

IROC at x=5

2)a) At each of the indicated points on the graph, is the instantaneous rate of change positive, negative, or zero?

A = positive
B = zero
C = negative



b) Estimate the instantaneous rate of change at points A and C.

$\frac{\Delta h}{\Delta t}\ \frac{16-12}{3-2} = \frac{4}{1}$

-4 m/s

$\frac{\Delta h}{\Delta t}\ \frac{13-7}{6-7}\quad \frac{6}{-1}$

= -6 m/s

c) Interpret the values in part b) for the situation represented by the graph.

When the distance changes by time it gives a velocity.

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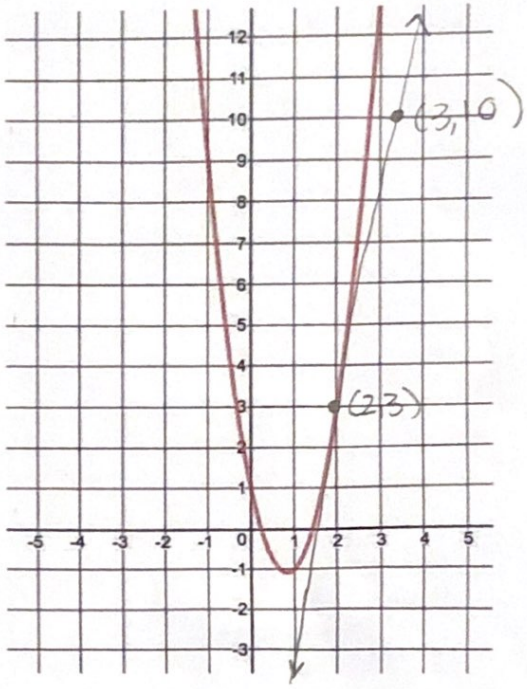
3) Use the graph of each function to estimate the instantaneous rate of change at $x = 2$ by drawing a tangent line and calculating it's slope.

a) $3x^2 - 5x + 1$



$m = \frac{\Delta y}{\Delta x} = \frac{10-3}{3-2} = 7$

$\frac{\Delta y}{\Delta x} \Big| \; x=2 \approx 7$

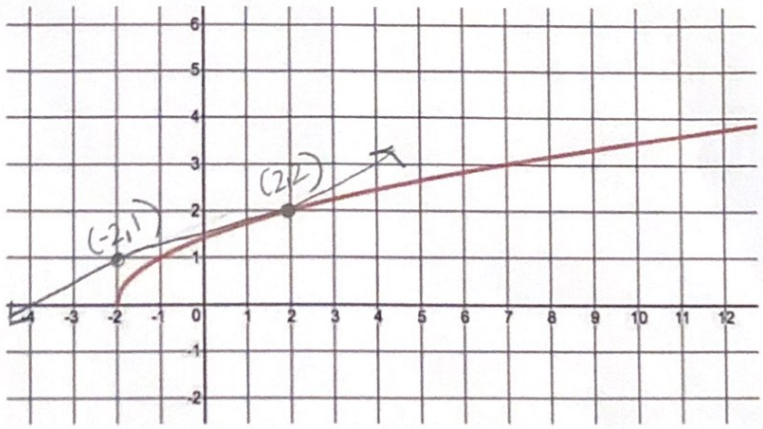
$3(2x) - 5 + 0$

$6x - 5 + 0$

$f(2) = 6(2) - 5$

$12 - 5 = 7$

b) $\sqrt{x+2}$



$m = \frac{2-1}{2-(-2)} = \frac{1}{4}$

$\frac{\Delta y}{\Delta x} \Big| \; x=2 \simeq \frac{1}{4}$

also 0.25

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4) Verify your answers from question #3 by calculating the LIMIT of the secant slopes as you approach $x = 2$.

a)

Interval	Δy	Δx	Slope of secant $= \frac{\Delta y}{\Delta x}$
$2 \le x \le 2.5$	$= f(2.5) - f(2)$ $= 7.25 - 3$ $= 4.25$	$= 2.5 - 2$ $= 0.5$	$= \frac{4.25}{0.5}$ $= 8.5$
$2 \le x \le 2.1$	$f(2.1) - f(2)$ $= 3.73 - 3$ $= 0.73$	$= 2.1 - 2$ $= 0.1$	$= \frac{0.73}{0.1}$ $= 7.3$
$2 \le x \le 2.01$	$f(2.01) - f(2)$ $= 3.0703 - 3$ $= 0.0703$	$= 2.01 - 2$ $= 0.01$	$= \frac{0.0703}{0.01}$ $= 7.03$
$2 \le x \le 2.001$	$f(2.001) - f(2)$ $= 3.007003 - 3$ $= 0.007003$	$2.001 - 2$ $= 0.001$	$= \frac{0.007003}{.001}$ $= 7.003$

$$\left.\frac{\Delta y}{\Delta x}\right|_{x=2} \approx 7$$

b)

Interval	Δy	Δx	Slope of secant $= \frac{\Delta y}{\Delta x}$
$2 \le x \le 2.5$	$f(2.5) - f(2)$ $= 2.121320344 - 2$ $= 0.121320344$	$= 2.5 - 2$ $= 0.5$	$\frac{0.121320344}{0.5}$ $= 0.242640687$
$2 \le x \le 2.1$	$f(2.1) - f(2)$ $= 2.024845673 - 2$ $= 0.024845673$	$= 2.1 - 2$ $= 0.1$	$= \frac{0.024845673}{0.1}$ 0.248456731
$2 \le x \le 2.01$	$f(2.01) - f(2)$ $= 2.002498439 - 2$ $= 0.002498439$	$= 2.01 - 2$ $= 0.01$	$= \frac{0.002498439}{0.01}$ $= 0.2498439$
$2 \le x \le 2.001$	$f(2.001) - f(2)$ $= 2.000249984 - 2$ $= 0.000249984$	$= 2.001 - 2$ $= 0.001$	$= \frac{0.000249984}{0.001}$ $= 0.249984$

$$\left.\frac{\Delta y}{\Delta x}\right|_{x=2} \approx 0.25$$