

- c. The least common denominator is $2(x - 2)$. Multiply both sides of the equation by $x(x - 2)$.

$$2(x - 2)\left(\frac{x}{x - 2}\right) = \left(\frac{5}{x - 2} - \frac{1}{2}\right)2(x - 2)$$

$$2x = 10 - (x - 2)$$

$$2x = 12 - x$$

$$3x = 12$$

$$x = 4$$

The solution is 4. The excluded value is 2.

Try It #5

Solve $\frac{-3}{2x + 1} = \frac{4}{3x + 1}$. State the excluded values.

Example 7 Solving a Rational Equation with Factored Denominators and Stating Excluded Values

Solve the rational equation after factoring the denominators: $\frac{2}{x + 1} - \frac{1}{x - 1} = \frac{2x}{x^2 - 1}$. State the excluded values.

Solution We must factor the denominator $x^2 - 1$. We recognize this as the difference of squares, and factor it as $(x - 1)(x + 1)$. Thus, the LCD that contains each denominator is $(x - 1)(x + 1)$. Multiply the whole equation by the LCD, cancel out the denominators, and solve the remaining equation.

$$(x - 1)(x + 1)\left(\frac{2}{x + 1} - \frac{1}{x - 1}\right) = \left(\frac{2x}{(x - 1)(x + 1)}\right)(x - 1)(x + 1)$$

$$2(x - 1) - 1(x + 1) = 2x$$

$$2x - 2 - x - 1 = 2x \quad \text{Distribute the negative sign.}$$

$$-3 - x = 0$$

$$-3 = x$$

The solution is -3 . The excluded values are 1 and -1 .

Try It #6

Solve the rational equation: $\frac{2}{x - 2} + \frac{1}{x + 1} = \frac{1}{x^2 - x - 2}$.

Finding a Linear Equation

Perhaps the most familiar form of a linear equation is the slope-intercept form, written as $y = mx + b$, where m = slope and b = y -intercept. Let us begin with the slope.

The Slope of a Line

The **slope** of a line refers to the ratio of the vertical change in y over the horizontal change in x between any two points on a line. It indicates the direction in which a line slants as well as its steepness. Slope is sometimes described as rise over run.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

If the slope is positive, the line slants to the right. If the slope is negative, the line slants to the left. As the slope increases, the line becomes steeper. Some examples are shown in **Figure 2**. The lines indicate the following slopes: $m = -3$, $m = 2$, and $m = \frac{1}{3}$.

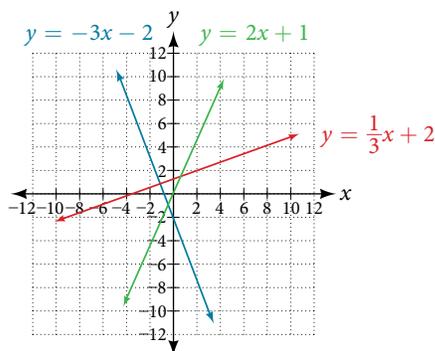


Figure 2

the slope of a line

The slope of a line, m , represents the change in y over the change in x . Given two points, (x_1, y_1) and (x_2, y_2) , the following formula determines the slope of a line containing these points:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Example 8 Finding the Slope of a Line Given Two Points

Find the slope of a line that passes through the points $(2, -1)$ and $(-5, 3)$.

Solution We substitute the y -values and the x -values into the formula.

$$\begin{aligned} m &= \frac{3 - (-1)}{-5 - 2} \\ &= \frac{4}{-7} \\ &= -\frac{4}{7} \end{aligned}$$

The slope is $-\frac{4}{7}$.

Analysis It does not matter which point is called (x_1, y_1) or (x_2, y_2) . As long as we are consistent with the order of the y terms and the order of the x terms in the numerator and denominator, the calculation will yield the same result.

Try It #7

Find the slope of the line that passes through the points $(-2, 6)$ and $(1, 4)$.

Example 9 Identifying the Slope and y -intercept of a Line Given an Equation

Identify the slope and y -intercept, given the equation $y = -\frac{3}{4}x - 4$.

Solution As the line is in $y = mx + b$ form, the given line has a slope of $m = -\frac{3}{4}$. The y -intercept is $b = -4$.

Analysis The y -intercept is the point at which the line crosses the y -axis. On the y -axis, $x = 0$. We can always identify the y -intercept when the line is in slope-intercept form, as it will always equal b . Or, just substitute $x = 0$ and solve for y .

The Point-Slope Formula

Given the slope and one point on a line, we can find the equation of the line using the point-slope formula.

$$y - y_1 = m(x - x_1)$$

This is an important formula, as it will be used in other areas of college algebra and often in calculus to find the equation of a tangent line. We need only one point and the slope of the line to use the formula. After substituting the slope and the coordinates of one point into the formula, we simplify it and write it in slope-intercept form.

the point-slope formula

Given one point and the slope, the point-slope formula will lead to the equation of a line:

$$y - y_1 = m(x - x_1)$$

Example 10 Finding the Equation of a Line Given the Slope and One Point

Write the equation of the line with slope $m = -3$ and passing through the point $(4, 8)$. Write the final equation in slope-intercept form.

Solution Using the point-slope formula, substitute -3 for m and the point $(4, 8)$ for (x_1, y_1) .

$$y - y_1 = m(x - x_1)$$

$$y - 8 = -3(x - 4)$$

$$y - 8 = -3x + 12$$

$$y = -3x + 20$$

Analysis Note that any point on the line can be used to find the equation. If done correctly, the same final equation will be obtained.

Try It #8

Given $m = 4$, find the equation of the line in slope-intercept form passing through the point $(2, 5)$.

Example 11 Finding the Equation of a Line Passing Through Two Given Points

Find the equation of the line passing through the points $(3, 4)$ and $(0, -3)$. Write the final equation in slope-intercept form.

Solution First, we calculate the slope using the slope formula and two points.

$$m = \frac{-3 - 4}{0 - 3}$$

$$= \frac{-7}{-3}$$

$$= \frac{7}{3}$$

Next, we use the point-slope formula with the slope of $\frac{7}{3}$, and either point. Let's pick the point $(3, 4)$ for (x_1, y_1) .

$$y - 4 = \frac{7}{3}(x - 3)$$

$$y - 4 = \frac{7}{3}x - 7 \quad \text{Distribute the } \frac{7}{3}.$$

$$y = \frac{7}{3}x - 3$$

In slope-intercept form, the equation is written as $y = \frac{7}{3}x - 3$.

Analysis To prove that either point can be used, let us use the second point $(0, -3)$ and see if we get the same equation.

$$y - (-3) = \frac{7}{3}(x - 0)$$

$$y + 3 = \frac{7}{3}x$$

$$y = \frac{7}{3}x - 3$$

We see that the same line will be obtained using either point. This makes sense because we used both points to calculate the slope.

Standard Form of a Line

Another way that we can represent the equation of a line is in standard form. Standard form is given as

$$Ax + By = C$$

where A , B , and C are integers. The x - and y -terms are on one side of the equal sign and the constant term is on the other side.

Example 12 Finding the Equation of a Line and Writing It in Standard Form

Find the equation of the line with $m = -6$ and passing through the point $(\frac{1}{4}, -2)$. Write the equation in standard form.

Solution We begin using the point-slope formula.

$$\begin{aligned} y - (-2) &= -6\left(x - \frac{1}{4}\right) \\ y + 2 &= -6x + \frac{3}{2} \end{aligned}$$

From here, we multiply through by 2, as no fractions are permitted in standard form, and then move both variables to the left side of the equal sign and move the constants to the right.

$$\begin{aligned} 2(y + 2) &= \left(-6x + \frac{3}{2}\right)2 \\ 2y + 4 &= -12x + 3 \\ 12x + 2y &= -1 \end{aligned}$$

This equation is now written in standard form.

Try It #9

Find the equation of the line in standard form with slope $m = -\frac{1}{3}$ and passing through the point $(1, \frac{1}{3})$.

Vertical and Horizontal Lines

The equations of vertical and horizontal lines do not require any of the preceding formulas, although we can use the formulas to prove that the equations are correct. The equation of a vertical line is given as

$$x = c$$

where c is a constant. The slope of a vertical line is undefined, and regardless of the y -value of any point on the line, the x -coordinate of the point will be c .

Suppose that we want to find the equation of a line containing the following points: $(-3, -5)$, $(-3, 1)$, $(-3, 3)$, and $(-3, 5)$. First, we will find the slope.

$$m = \frac{5 - 3}{-3 - (-3)} = \frac{2}{0}$$

Zero in the denominator means that the slope is undefined and, therefore, we cannot use the point-slope formula. However, we can plot the points. Notice that all of the x -coordinates are the same and we find a vertical line through $x = -3$. See **Figure 3**.

The equation of a horizontal line is given as

$$y = c$$

where c is a constant. The slope of a horizontal line is zero, and for any x -value of a point on the line, the y -coordinate will be c .

Suppose we want to find the equation of a line that contains the following set of points: $(-2, -2)$, $(0, -2)$, $(3, -2)$, and $(5, -2)$. We can use the point-slope formula. First, we find the slope using any two points on the line.

$$\begin{aligned} m &= \frac{-2 - (-2)}{0 - (-2)} \\ &= \frac{0}{2} \\ &= 0 \end{aligned}$$

Use any point for (x_1, y_1) in the formula, or use the y -intercept.

$$\begin{aligned} y - (-2) &= 0(x - 3) \\ y + 2 &= 0 \\ y &= -2 \end{aligned}$$

The graph is a horizontal line through $y = -2$. Notice that all of the y -coordinates are the same. See **Figure 3**.

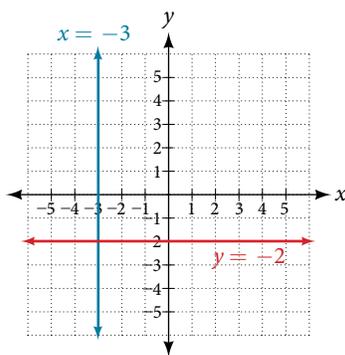


Figure 3 The line $x = -3$ is a vertical line. The line $y = -2$ is a horizontal line.

Example 13 Finding the Equation of a Line Passing Through the Given Points

Find the equation of the line passing through the given points: $(1, -3)$ and $(1, 4)$.

Solution The x -coordinate of both points is 1. Therefore, we have a vertical line, $x = 1$.

Try It #10

Find the equation of the line passing through $(-5, 2)$ and $(2, 2)$.

Determining Whether Graphs of Lines are Parallel or Perpendicular

Parallel lines have the same slope and different y -intercepts. Lines that are parallel to each other will never intersect. For example, **Figure 4** shows the graphs of various lines with the same slope, $m = 2$.

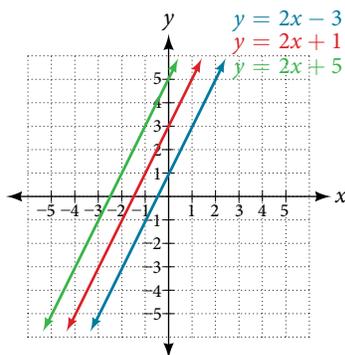


Figure 4 Parallel lines

All of the lines shown in the graph are parallel because they have the same slope and different y -intercepts.

Lines that are perpendicular intersect to form a 90° -angle. The slope of one line is the negative reciprocal of the other.

We can show that two lines are perpendicular if the product of the two slopes is -1 : $m_1 \cdot m_2 = -1$. For example, **Figure 5** shows the graph of two perpendicular lines. One line has a slope of 3; the other line has a slope of $-\frac{1}{3}$.

$$m_1 \cdot m_2 = -1$$

$$3 \cdot \left(-\frac{1}{3}\right) = -1$$

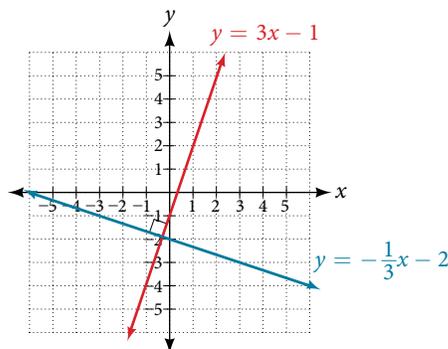


Figure 5 Perpendicular lines

Example 14 Graphing Two Equations, and Determining Whether the Lines are Parallel, Perpendicular, or Neither

Graph the equations of the given lines, and state whether they are parallel, perpendicular, or neither: $3y = -4x + 3$ and $3x - 4y = 8$.

Solution The first thing we want to do is rewrite the equations so that both equations are in slope-intercept form.

First equation:

$$3y = -4x + 3$$

$$y = -\frac{4}{3}x + 1$$

Second equation:

$$3x - 4y = 8$$

$$-4y = -3x + 8$$

$$y = \frac{3}{4}$$

See the graph of both lines in **Figure 6**.

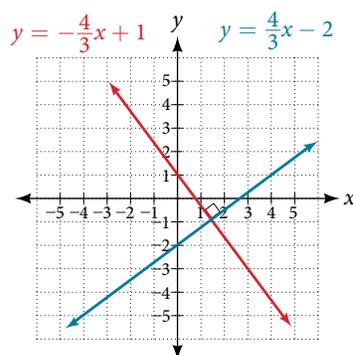


Figure 6

From the graph, we can see that the lines appear perpendicular, but we must compare the slopes.

$$m_1 = -\frac{4}{3}$$

$$m_2 = \frac{3}{4}$$

$$m_1 \cdot m_2 = \left(-\frac{4}{3}\right)\left(\frac{3}{4}\right) = -1$$

The slopes are negative reciprocals of each other, confirming that the lines are perpendicular.

Try It #11

Graph the two lines and determine whether they are parallel, perpendicular, or neither: $2y - x = 10$ and $2y = x + 4$.

Writing the Equations of Lines Parallel or Perpendicular to a Given Line

As we have learned, determining whether two lines are parallel or perpendicular is a matter of finding the slopes. To write the equation of a line parallel or perpendicular to another line, we follow the same principles as we do for finding the equation of any line. After finding the slope, use the point-slope formula to write the equation of the new line.

How To...

Given an equation for a line, write the equation of a line parallel or perpendicular to it.

1. Find the slope of the given line. The easiest way to do this is to write the equation in slope-intercept form.
2. Use the slope and the given point with the point-slope formula.
3. Simplify the line to slope-intercept form and compare the equation to the given line.

Example 15 Writing the Equation of a Line Parallel to a Given Line Passing Through a Given Point

Write the equation of line parallel to a $5x + 3y = 1$ and passing through the point $(3, 5)$.

Solution First, we will write the equation in slope-intercept form to find the slope.

$$\begin{aligned} 5x + 3y &= 1 \\ 3y &= 5x + 1 \\ y &= -\frac{5}{3}x + \frac{1}{3} \end{aligned}$$

The slope is $m = -\frac{5}{3}$. The y -intercept is $\frac{1}{3}$, but that really does not enter into our problem, as the only thing we need for two lines to be parallel is the same slope. The one exception is that if the y -intercepts are the same, then the two lines are the same line. The next step is to use this slope and the given point with the point-slope formula.

$$\begin{aligned} y - 5 &= -\frac{5}{3}(x - 3) \\ y - 5 &= -\frac{5}{3}x + 5 \\ y &= -\frac{5}{3}x + 10 \end{aligned}$$

The equation of the line is $y = -\frac{5}{3}x + 10$. See **Figure 7**.

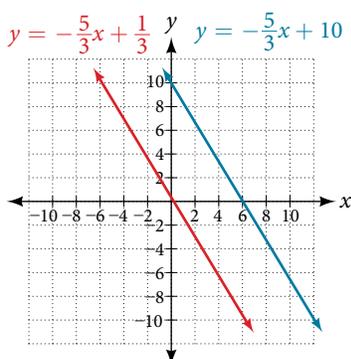


Figure 7

Try It #12

Find the equation of the line parallel to $5x = 7 + y$ and passing through the point $(-1, -2)$.

Example 16 Finding the Equation of a Line Perpendicular to a Given Line Passing Through a Given Point

Find the equation of the line perpendicular to $5x - 3y + 4 = 0$ and passes through the point $(-4, 1)$.

Solution The first step is to write the equation in slope-intercept form.

$$\begin{aligned}5x - 3y + 4 &= 0 \\-3y &= -5x - 4 \\y &= \frac{5}{3}x + \frac{4}{3}\end{aligned}$$

We see that the slope is $m = \frac{5}{3}$. This means that the slope of the line perpendicular to the given line is the negative reciprocal, or $-\frac{3}{5}$. Next, we use the point-slope formula with this new slope and the given point.

$$\begin{aligned}y - 1 &= -\frac{3}{5}(x - (-4)) \\y - 1 &= -\frac{3}{5}x - \frac{12}{5} \\y &= -\frac{3}{5}x - \frac{12}{5} + \frac{5}{5} \\y &= -\frac{3}{5}x - \frac{7}{5}\end{aligned}$$

Access these online resources for additional instruction and practice with linear equations.

- Solving rational equations (<http://openstaxcollege.org/l/rationaleqs>)
- Equation of a line given two points (<http://openstaxcollege.org/l/twopointsline>)
- Finding the equation of a line perpendicular to another line through a given point (<http://openstaxcollege.org/l/findperpline>)
- Finding the equation of a line parallel to another line through a given point (<http://openstaxcollege.org/l/findparaline>)

Example 4 Solving a Perimeter Problem

The perimeter of a rectangular outdoor patio is 54 ft. The length is 3 ft greater than the width. What are the dimensions of the patio?

Solution The perimeter formula is standard: $P = 2L + 2W$. We have two unknown quantities, length and width. However, we can write the length in terms of the width as $L = W + 3$. Substitute the perimeter value and the expression for length into the formula. It is often helpful to make a sketch and label the sides as in **Figure 3**.

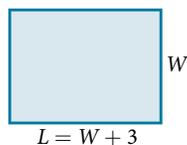


Figure 3

Now we can solve for the width and then calculate the length.

$$\begin{aligned} P &= 2L + 2W \\ 54 &= 2(W + 3) + 2W \\ 54 &= 2W + 6 + 2W \\ 54 &= 4W + 6 \\ 48 &= 4W \\ 12 &= W \\ (12 + 3) &= L \\ 15 &= L \end{aligned}$$

The dimensions are $L = 15$ ft and $W = 12$ ft.

Try It #4

Find the dimensions of a rectangle given that the perimeter is 110 cm and the length is 1 cm more than twice the width.

Example 5 Solving an Area Problem

The perimeter of a tablet of graph paper is 48 in.². The length is 6 in. more than the width. Find the area of the graph paper.

Solution The standard formula for area is $A = LW$; however, we will solve the problem using the perimeter formula. The reason we use the perimeter formula is because we know enough information about the perimeter that the formula will allow us to solve for one of the unknowns. As both perimeter and area use length and width as dimensions, they are often used together to solve a problem such as this one.

We know that the length is 6 in. more than the width, so we can write length as $L = W + 6$. Substitute the value of the perimeter and the expression for length into the perimeter formula and find the length.

$$\begin{aligned} P &= 2L + 2W \\ 48 &= 2(W + 6) + 2W \\ 48 &= 2W + 12 + 2W \\ 48 &= 4W + 12 \\ 36 &= 4W \\ 9 &= W \\ (9 + 6) &= L \\ 15 &= L \end{aligned}$$

Now, we find the area given the dimensions of $L = 15$ in. and $W = 9$ in.

$$\begin{aligned}A &= LW \\A &= 15(9) \\&= 135 \text{ in.}^2\end{aligned}$$

The area is 135 in.^2 .

Try It #5

A game room has a perimeter of 70 ft. The length is five more than twice the width. How many ft^2 of new carpeting should be ordered?

Example 6 Solving a Volume Problem

Find the dimensions of a shipping box given that the length is twice the width, the height is 8 inches, and the volume is $1,600 \text{ in}^3$.

Solution The formula for the volume of a box is given as $V = LWH$, the product of length, width, and height. We are given that $L = 2W$, and $H = 8$. The volume is 1,600 cubic inches.

$$\begin{aligned}V &= LWH \\1,600 &= (2W)W(8) \\1,600 &= 16W^2 \\100 &= W^2 \\10 &= W\end{aligned}$$

The dimensions are $L = 20$ in., $W = 10$ in., and $H = 8$ in.

Analysis Note that the square root of W^2 would result in a positive and a negative value. However, because we are describing width, we can use only the positive result.

Access these online resources for additional instruction and practice with models and applications of linear equations.

- [Problem Solving Using Linear Equations \(http://openstaxcollege.org//lineqprobsolve\)](http://openstaxcollege.org//lineqprobsolve)
- [Problem Solving Using Equations \(http://openstaxcollege.org//equationprsolve\)](http://openstaxcollege.org//equationprsolve)
- [Finding the Dimensions and Area Given the Perimeter \(http://openstaxcollege.org//permareasolve\)](http://openstaxcollege.org//permareasolve)
- [Find the Distance Between the Cities Using the distance = rate * time formula \(http://openstaxcollege.org//ratetimesolve\)](http://openstaxcollege.org//ratetimesolve)
- [Linear Equation Application \(Write a cost equation\) \(http://openstaxcollege.org//lineqappl\)](http://openstaxcollege.org//lineqappl)

LEARNING OBJECTIVES

In this section, you will:

- Solve algebraic and numeric variation problems.
- Solve application problems including direct, inverse, and/or joint variation.

5.8 MODELING USING VARIATION

A used-car company has just offered their best candidate, Nicole, a position in sales. The position offers 16% commission on her sales. Her earnings depend on the amount of her sales. For instance, if she sells a vehicle for \$4,600, she will earn \$736. She wants to evaluate the offer, but she is not sure how. In this section, we will look at relationships, such as this one, between earnings, sales, and commission rate.

Solving Direct Variation Problems

In the example above, Nicole's earnings can be found by multiplying her sales by her commission. The formula $e = 0.16s$ tells us her earnings, e , come from the product of 0.16, her commission, and the sale price of the vehicle. If we create a table, we observe that as the sales price increases, the earnings increase as well, which should be intuitive. See **Table 1**.

s , sales price	$e = 0.16s$	Interpretation
\$4,600	$e = 0.16(4,600) = 736$	A sale of a \$4,600 vehicle results in \$736 earnings.
\$9,200	$e = 0.16(9,200) = 1,472$	A sale of a \$9,200 vehicle results in \$1,472 earnings.
\$18,400	$e = 0.16(18,400) = 2,944$	A sale of a \$18,400 vehicle results in \$2,944 earnings.

Table 1

Notice that earnings are a multiple of sales. As sales increase, earnings increase in a predictable way. Double the sales of the vehicle from \$4,600 to \$9,200, and we double the earnings from \$736 to \$1,472. As the input increases, the output increases as a multiple of the input. A relationship in which one quantity is a constant multiplied by another quantity is called **direct variation**. Each variable in this type of relationship **varies directly** with the other.

Figure 1 represents the data for Nicole's potential earnings. We say that earnings vary directly with the sales price of the car. The formula $y = kx^n$ is used for direct variation. The value k is a nonzero constant greater than zero and is called the **constant of variation**. In this case, $k = 0.16$ and $n = 1$. We saw functions like this one when we discussed power functions.

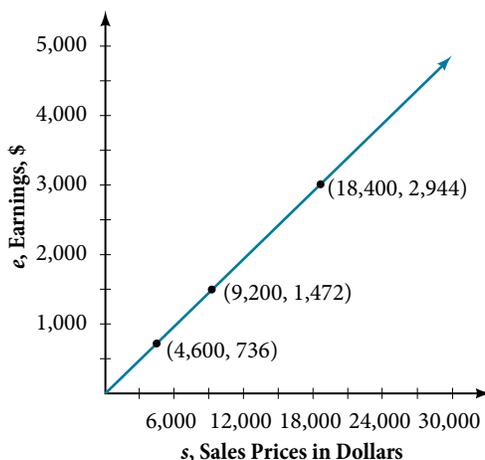


Figure 1

direct variation

If x and y are related by an equation of the form

$$y = kx^n$$

then we say that the relationship is **direct variation** and y **varies directly** with, or is proportional to, the n th power of x . In direct variation relationships, there is a nonzero constant ratio $k = \frac{y}{x^n}$, where k is called the **constant of variation**, which help defines the relationship between the variables.

How To...

Given a description of a direct variation problem, solve for an unknown.

1. Identify the input, x , and the output, y .
2. Determine the constant of variation. You may need to divide y by the specified power of x to determine the constant of variation.
3. Use the constant of variation to write an equation for the relationship.
4. Substitute known values into the equation to find the unknown.

Example 1 Solving a Direct Variation Problem

The quantity y varies directly with the cube of x . If $y = 25$ when $x = 2$, find y when x is 6.

Solution The general formula for direct variation with a cube is $y = kx^3$. The constant can be found by dividing y by the cube of x .

$$\begin{aligned} k &= \frac{y}{x^3} \\ &= \frac{25}{2^3} \\ &= \frac{25}{8} \end{aligned}$$

Now use the constant to write an equation that represents this relationship.

$$y = \frac{25}{8}x^3$$

Substitute $x = 6$ and solve for y .

$$\begin{aligned} y &= \frac{25}{8}(6)^3 \\ &= 675 \end{aligned}$$

Analysis The graph of this equation is a simple cubic, as shown in **Figure 2**.

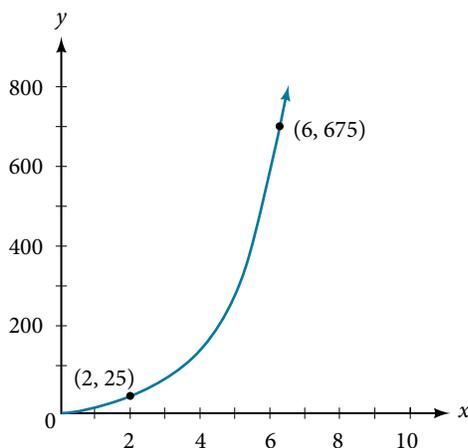


Figure 2

Q & A...

Do the graphs of all direct variation equations look like Example 1?

No. Direct variation equations are power functions—they may be linear, quadratic, cubic, quartic, radical, etc. But all of the graphs pass through (0,0).

Try It #1

The quantity y varies directly with the square of x . If $y = 24$ when $x = 3$, find y when x is 4.

Solving Inverse Variation Problems

Water temperature in an ocean varies inversely to the water's depth. Between the depths of 250 feet and 500 feet, the formula $T = \frac{14,000}{d}$ gives us the temperature in degrees Fahrenheit at a depth in feet below Earth's surface. Consider the Atlantic Ocean, which covers 22% of Earth's surface. At a certain location, at the depth of 500 feet, the temperature may be 28°F. If we create **Table 2**, we observe that, as the depth increases, the water temperature decreases.

d , depth	$T = \frac{14,000}{d}$	Interpretation
500 ft	$\frac{14,000}{500} = 28$	At a depth of 500 ft, the water temperature is 28° F.
1,000 ft	$\frac{14,000}{1,000} = 14$	At a depth of 1,000 ft, the water temperature is 14° F.
2,000 ft	$\frac{14,000}{2,000} = 7$	At a depth of 2,000 ft, the water temperature is 7° F.

Table 2

We notice in the relationship between these variables that, as one quantity increases, the other decreases. The two quantities are said to be **inversely proportional** and each term **varies inversely** with the other. Inversely proportional relationships are also called **inverse variations**.

For our example, **Figure 3** depicts the inverse variation. We say the water temperature varies inversely with the depth of the water because, as the depth increases, the temperature decreases. The formula $y = \frac{k}{x}$ for inverse variation in this case uses $k = 14,000$.

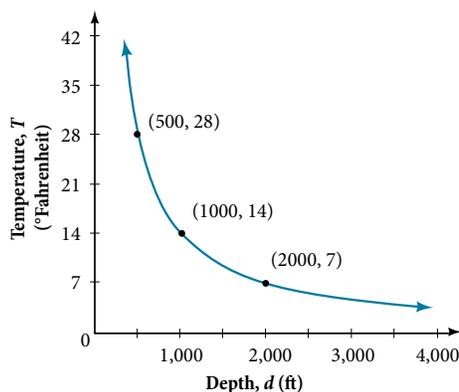


Figure 3

inverse variation

If x and y are related by an equation of the form

$$y = \frac{k}{x^n}$$

where k is a nonzero constant, then we say that y **varies inversely** with the n th power of x . In **inversely proportional** relationships, or **inverse variations**, there is a constant multiple $k = x^n y$.

Example 2 Writing a Formula for an Inversely Proportional Relationship

A tourist plans to drive 100 miles. Find a formula for the time the trip will take as a function of the speed the tourist drives.

Solution Recall that multiplying speed by time gives distance. If we let t represent the drive time in hours, and v represent the velocity (speed or rate) at which the tourist drives, then $vt = \text{distance}$. Because the distance is fixed at 100 miles, $vt = 100$ so $t = \frac{100}{v}$. Because time is a function of velocity, we can write $t(v)$.

$$\begin{aligned} t(v) &= \frac{100}{v} \\ &= 100v^{-1} \end{aligned}$$

We can see that the constant of variation is 100 and, although we can write the relationship using the negative exponent, it is more common to see it written as a fraction. We say that the time varies inversely with velocity.

How To...

Given a description of an indirect variation problem, solve for an unknown.

1. Identify the input, x , and the output, y .
2. Determine the constant of variation. You may need to multiply y by the specified power of x to determine the constant of variation.
3. Use the constant of variation to write an equation for the relationship.
4. Substitute known values into the equation to find the unknown.

Example 3 Solving an Inverse Variation Problem

A quantity y varies inversely with the cube of x . If $y = 25$ when $x = 2$, find y when x is 6.

Solution The general formula for inverse variation with a cube is $y = \frac{k}{x^3}$. The constant can be found by multiplying y by the cube of x .

$$\begin{aligned} k &= x^3 y \\ &= 2^3 \cdot 25 \\ &= 200 \end{aligned}$$

Now we use the constant to write an equation that represents this relationship.

$$\begin{aligned} y &= \frac{k}{x^3}, k = 200 \\ y &= \frac{200}{x^3} \end{aligned}$$

Substitute $x = 6$ and solve for y .

$$\begin{aligned} y &= \frac{200}{6^3} \\ &= \frac{25}{27} \end{aligned}$$

Analysis The graph of this equation is a rational function, as shown in **Figure 4**.

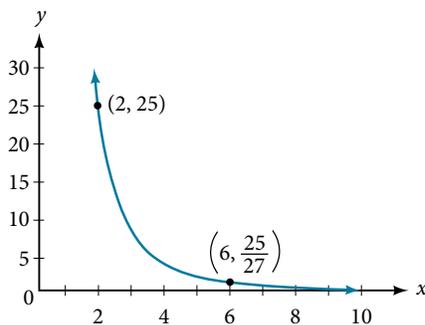


Figure 4

Try It #2

A quantity y varies inversely with the square of x . If $y = 8$ when $x = 3$, find y when x is 4.

Solving Problems Involving Joint Variation

Many situations are more complicated than a basic direct variation or inverse variation model. One variable often depends on multiple other variables. When a variable is dependent on the product or quotient of two or more variables, this is called **joint variation**. For example, the cost of busing students for each school trip varies with the number of students attending and the distance from the school. The variable c , cost, varies jointly with the number of students, n , and the distance, d .

joint variation

Joint variation occurs when a variable varies directly or inversely with multiple variables.

For instance, if x varies directly with both y and z , we have $x = kyz$. If x varies directly with y and inversely with z , we have $x = \frac{ky}{z}$. Notice that we only use one constant in a joint variation equation.

Example 4 Solving Problems Involving Joint Variation

A quantity x varies directly with the square of y and inversely with the cube root of z . If $x = 6$ when $y = 2$ and $z = 8$, find x when $y = 1$ and $z = 27$.

Solution Begin by writing an equation to show the relationship between the variables.

$$x = \frac{ky^2}{\sqrt[3]{z}}$$

Substitute $x = 6$, $y = 2$, and $z = 8$ to find the value of the constant k .

$$6 = \frac{k2^2}{\sqrt[3]{8}}$$

$$6 = \frac{4k}{2}$$

$$3 = k$$

Now we can substitute the value of the constant into the equation for the relationship.

$$x = \frac{3y^2}{\sqrt[3]{z}}$$

To find x when $y = 1$ and $z = 27$, we will substitute values for y and z into our equation.

$$\begin{aligned} x &= \frac{3(1)^2}{\sqrt[3]{27}} \\ &= 1 \end{aligned}$$

Try It #3

A quantity x varies directly with the square of y and inversely with z . If $x = 40$ when $y = 4$ and $z = 2$, find x when $y = 10$ and $z = 25$.

Access these online resources for additional instruction and practice with direct and inverse variation.

- [Direct Variation \(http://openstaxcollege.org/l/directvariation\)](http://openstaxcollege.org/l/directvariation)
- [Inverse Variation \(http://openstaxcollege.org/l/inversevariatio\)](http://openstaxcollege.org/l/inversevariatio)
- [Direct and Inverse Variation \(http://openstaxcollege.org/l/directinverse\)](http://openstaxcollege.org/l/directinverse)