

radical equations

An equation containing terms with a variable in the radicand is called a **radical equation**.

How To...

Given a radical equation, solve it.

1. Isolate the radical expression on one side of the equal sign. Put all remaining terms on the other side.
2. If the radical is a square root, then square both sides of the equation. If it is a cube root, then raise both sides of the equation to the third power. In other words, for an n th root radical, raise both sides to the n th power. Doing so eliminates the radical symbol.
3. Solve the remaining equation.
4. If a radical term still remains, repeat steps 1–2.
5. Confirm solutions by substituting them into the original equation.

Example 6 Solving an Equation with One Radical

Solve $\sqrt{15 - 2x} = x$.

Solution The radical is already isolated on the left side of the equal side, so proceed to square both sides.

$$\begin{aligned}\sqrt{15 - 2x} &= x \\ (\sqrt{15 - 2x})^2 &= (x)^2 \\ 15 - 2x &= x^2\end{aligned}$$

We see that the remaining equation is a quadratic. Set it equal to zero and solve.

$$\begin{aligned}0 &= x^2 + 2x - 15 \\ 0 &= (x + 5)(x - 3) \\ 0 &= (x + 5) \quad \text{or} \quad 0 = (x - 3) \\ -5 &= x \quad \quad \quad \text{or} \quad 3 = x\end{aligned}$$

The proposed solutions are -5 and 3 . Let us check each solution back in the original equation. First, check -5 .

$$\begin{aligned}\sqrt{15 - 2x} &= x \\ \sqrt{15 - 2(-5)} &= -5 \\ \sqrt{25} &= -5 \\ 5 &\neq -5\end{aligned}$$

This is an extraneous solution. While no mistake was made solving the equation, we found a solution that does not satisfy the original equation.

Check 3 .

$$\begin{aligned}\sqrt{15 - 2x} &= x \\ \sqrt{15 - 2(3)} &= 3 \\ \sqrt{9} &= 3 \\ 3 &= 3\end{aligned}$$

The solution is 3 .

Try It #5

Solve the radical equation: $\sqrt{x + 3} = 3x - 1$

Example 7 Solving a Radical Equation Containing Two RadicalsSolve $\sqrt{2x+3} + \sqrt{x-2} = 4$.**Solution** As this equation contains two radicals, we isolate one radical, eliminate it, and then isolate the second radical.

$$\begin{aligned}\sqrt{2x+3} + \sqrt{x-2} &= 4 \\ \sqrt{2x+3} &= 4 - \sqrt{x-2} && \text{Subtract } \sqrt{x-2} \text{ from both sides.} \\ (\sqrt{2x+3})^2 &= (4 - \sqrt{x-2})^2 && \text{Square both sides.}\end{aligned}$$

Use the perfect square formula to expand the right side: $(a - b)^2 = a^2 - 2ab + b^2$.

$$\begin{aligned}2x + 3 &= (4)^2 - 2(4)\sqrt{x-2} + (\sqrt{x-2})^2 \\ 2x + 3 &= 16 - 8\sqrt{x-2} + (x-2) \\ 2x + 3 &= 14 + x - 8\sqrt{x-2} && \text{Combine like terms.} \\ x - 11 &= -8\sqrt{x-2} && \text{Isolate the second radical.} \\ (x - 11)^2 &= (-8\sqrt{x-2})^2 && \text{Square both sides.} \\ x^2 - 22x + 121 &= 64(x-2)\end{aligned}$$

Now that both radicals have been eliminated, set the quadratic equal to zero and solve.

$$\begin{aligned}x^2 - 22x + 121 &= 64x - 128 \\ x^2 - 86x + 249 &= 0 \\ (x - 3)(x - 83) &= 0 && \text{Factor and solve.} \\ x - 3 = 0 \quad \text{or} \quad x - 83 = 0 \\ x = 3 \quad \text{or} \quad x = 83\end{aligned}$$

The proposed solutions are 3 and 83. Check each solution in the original equation.

$$\begin{aligned}\sqrt{2x+3} + \sqrt{x-2} &= 4 \\ \sqrt{2x+3} &= 4 - \sqrt{x-2} \\ \sqrt{2(3)+3} &= 4 - \sqrt{(3)-2} \\ \sqrt{9} &= 4 - \sqrt{1} \\ 3 &= 3\end{aligned}$$

One solution is 3.

Check 83.

$$\begin{aligned}\sqrt{2x+3} + \sqrt{x-2} &= 4 \\ \sqrt{2x+3} &= 4 - \sqrt{x-2} \\ \sqrt{2(83)+3} &= 4 - \sqrt{(83)-2} \\ \sqrt{169} &= 4 - \sqrt{81} \\ 13 &\neq -5\end{aligned}$$

The only solution is 3. We see that 83 is an extraneous solution.

Try It #6Solve the equation with two radicals: $\sqrt{3x+7} + \sqrt{x+2} = 1$.

Example 9 Solving a Fourth-degree Equation in Quadratic Form

Solve this fourth-degree equation: $3x^4 - 2x^2 - 1 = 0$.

Solution This equation fits the main criteria, that the power on the leading term is double the power on the middle term. Next, we will make a substitution for the variable term in the middle. Let $u = x^2$. Rewrite the equation in u .

$$3u^2 - 2u - 1 = 0$$

Now solve the quadratic.

$$3u^2 - 2u - 1 = 0$$

$$(3u + 1)(u - 1) = 0$$

Solve each factor and replace the original term for u .

$$3u + 1 = 0$$

$$3u = -1$$

$$u = -\frac{1}{3}$$

$$x^2 = -\frac{1}{3}$$

$$x = \pm i\sqrt{\frac{1}{3}}$$

$$u - 1 = 0$$

$$u = 1$$

$$x^2 = 1$$

$$x = \pm 1$$

The solutions are $\pm i\sqrt{\frac{1}{3}}$ and ± 1 .

Try It #8

Solve using substitution: $x^4 - 8x^2 - 9 = 0$.

Example 10 Solving an Equation in Quadratic Form Containing a Binomial

Solve the equation in quadratic form: $(x + 2)^2 + 11(x + 2) - 12 = 0$.

Solution This equation contains a binomial in place of the single variable. The tendency is to expand what is presented. However, recognizing that it fits the criteria for being in quadratic form makes all the difference in the solving process. First, make a substitution, letting $u = x + 2$. Then rewrite the equation in u .

$$u^2 + 11u - 12 = 0$$

$$(u + 12)(u - 1) = 0$$

Solve using the zero-factor property and then replace u with the original expression.

$$u + 12 = 0$$

$$u = -12$$

$$x + 2 = -12$$

$$x = -14$$

The second factor results in

$$u - 1 = 0$$

$$u = 1$$

$$x + 2 = 1$$

$$x = -1$$

We have two solutions: -14 and -1 .

Try It #9

Solve: $(x - 5)^2 - 4(x - 5) - 21 = 0$.