Nyimbi Brantley

1 Graph

2 $y=x^2+11x-15$

$y'=2x+11$

slope $=y'_{x=1}=2\cdot1+11=13$

slope $=13$ tangent equation at $P(1,-3)$ is

$y=3=13(x-1)$

$y=13x-16$

3.

4.

✓5 Graph

6

7. $\frac{d}{dy}(2xy - y^2) = \frac{d}{dx}(1)$

$2x\frac{d}{dx}[y] - 2y\frac{d}{dx}[y] + 2y$

$2xy' - 2yy' + 2y = 0$

$y' = -\frac{y}{x-y}$

$\boxed{y' = \frac{dy}{dx} = -\frac{y}{x-y}}$

8 Graph

10 $2x^3 - 3x^2 - 12x = 0$

$x(2x^2 - 3x - 12) = 0$

One y the root is $x = 0$

$2x^2 - 3x - 12 = 0$ $\quad x = \frac{3 + \sqrt{9+96}}{4} = \frac{3 \pm}{4}$

$\frac{3+\sqrt{105}}{4}, \frac{\sqrt{105}-3}{4}$ $\quad f'(x) - 6x^2 - 6x - 12$

critical point $\quad 6u^2 - 6y - 12 = 0 \qquad u = 2$ and $x = -1$

$u^2 - u - 2 = 0$

$u^2 - 2u + u - 2 = 0$

$u(u-2) + 1(u-2) = 0$

inflection point : $f''(u) = 0 \qquad u = \frac{1}{2}$

$12u - 6 = 0$

$f(\frac{1}{2}) = 2x(\frac{1}{2})^3 - 3(\frac{1}{2})^2 - 12 \times \frac{1}{2} = -\frac{13}{2}$

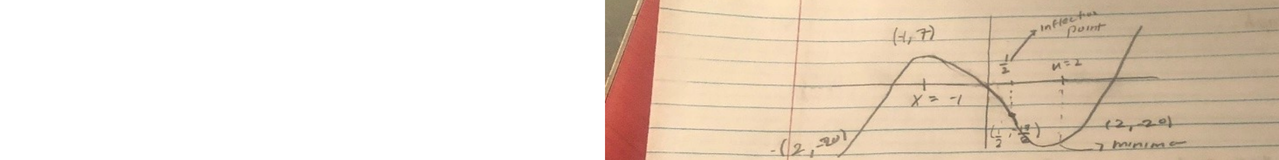
critical points = $u = -1$, and $x = 2$. So, the maxima and minima

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1)$

$= -2 + 3 + 12 = 7$

$f(2) = 2(2^3) - 3 \cdot 2^2 - 12(2)$

$= 16 - 12 - 24 = -20$



9 Graph

11. Given $g(x) = -x^2 + 11x - 24$, $3 \le x \le 8$
To find absolute extreme value. First we have to find all the critical points between 3 and 8

$g'(x) = 0$ $\quad x = \frac{11}{2}$ lies between 3 and 8
$-2x + 11 = 0$ $\quad g''(x) = -2$
$2x = 11$ $\quad g''(x)$ is negative

Since $g(x)$ has maximum value. The maximum value of $g(x)$ at $x = \frac{11}{2}$ is $g(x) = -\left(\frac{11}{2}\right)^2 + 11\left(\frac{11}{2}\right) - 24$

$= -\frac{121}{4} + \frac{121}{2} - 24$
$= -\frac{121}{4} - 24$
$= \frac{+121 - 24(4)}{4}$ $\quad = \frac{-121 - 96}{4}$
$g(x) = \frac{25}{4}$

The absolute minimum occurs at $x = 3$ and $x = 8$
put $x = 3$ in $g(x)$ $\quad g(x) = -9 + 33 - 24 = 0$
put $x = 8$ in $g(x)$ $\quad g(x) = -64 + 88 - 24 = 0$
absolute maximum is $\frac{25}{4}$ at $x = \frac{11}{2}$ and
absolute minimum is 0 at 8 and 0 at 3

12 Graph

13. at $x=0$ and $y=1$ to find the slope of the tangent line at $x=0$ and $y=1$

$= \frac{11}{4}$

$y - y_1 = m(x - x_1)$

$y - (1) = \frac{11}{4} \cdot (x - (0))$

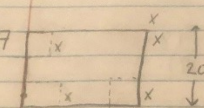
$y = \frac{11}{4} + x \; 1$

14 Graph

15

15 Graph

16

17 

The box formed will have
length, $l = 20-2x$
breadth, $b = 20-2x$
height, $h = x$

volume $= V = lbh = (20-2x)^2 \cdot x$
or $V = (400 + 4x^2 - 80x)x$ $V = 4x^3 - 80x^2 + 4w x$
for V to be maximum $\frac{dv}{dn} = 0$.
$\frac{dv}{dn} = 12n^2 - 160n + 4w = 0$
$3n^2 - 40n + 1w = 0$
$3n^2 - 30n - 10n + 1w = 0$
$3n(n-10) - 10(n-10) 20$
$(3n-10)(n-10) = 0 \Rightarrow n = 10, \frac{10}{3}$

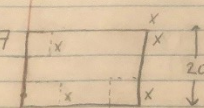
$n = 10$, $l = 0$, $b = 0$, not possible
$n = \frac{10}{3}$, dimensions are, length $l = 20 - \frac{20}{3} = \frac{40}{3}$ in
$b = \frac{40}{3}$ in $h = \frac{10}{3}$ in
volume $= \left(\frac{40}{3}\right)^2 \times \frac{10}{3} = \frac{1600}{27}$ in^3

18 $\lim_{x \to 0} f(n) =$ not defined
since $\lim_{x \to 0^-} f(x) = -1$ and $\lim_{x \to 0^+} f(x) = 1$

19 $-3x^2 + 3x + 6$
$f''(x) = -6x + 3$
$-3x^2 + 3x + 6 = 0$
$-3(x-2)(x+1) = 0$
$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$
$x - 2 = 0$
$x + 1 = 0$
$x = 2$
$x = -1$

$(x-2)(x+1) = 0$ true
$x = 2, -1$

17 

$\updownarrow$ 20

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$3n^2 - 40n + 1w = 0$

$3n^2 - 30n - 10n + 1w = 0$

$3n(n-10) - 10(n-10) 20$

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$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x - 2 = 0$

$x + 1 = 0$

$x = 2$

$x = -1$

$(x-2)(x+1) = 0$ true

$x = 2, -1$

20

21 $\frac{d}{dx}(x^3+3x^2y+y^3)=\frac{d}{dx}(0)$

$3x^2+3x^2\frac{d}{dx}[y]+6xy+3y^2\frac{d}{dx}[y]=0$

$3x^2+3x^2y'+6xy+3y^2y'=0$

$y'=-x\frac{(x+2y)}{x^2+y^2}$

22 $\sqrt{\frac{\lim_{x\to\infty} 16+0-3\cdot 0}{\lim_{x\to\infty} 1-8\cdot 0-9\cdot 0}}$ $\sqrt{\frac{16+0-3\cdot 0}{1-8\cdot 0-9\cdot 0}}=4$

23 $\frac{5}{x}+5\sec(x)$ with respect to x is

$\frac{d}{dx}\left[\frac{5}{x}\right]+\frac{d}{dx}[5\sec(x)]$

$\frac{d}{dx}\left[\frac{5}{x}\right]+\frac{d}{dx}[5\sec(x)]$

$\frac{d}{dx}\left[\frac{5}{x}\right]$ $-5x^{-2}+\frac{d}{dx}[\sec(x)]$

$\frac{d}{dx}[5\sec(x)]$

$-5x^{-2}+5\sec(x)\tan(x)$

$-5x^{-2}+\frac{d}{dx}[5\sec(x)]$

$\frac{d}{dx}[5\sec(x)]$

$-5x^{-2}+5\sec(x)\tan(x)$

$-5\frac{1}{x^2}+5\sec(x)\tan(x)$

$5\sec(x)\tan(x)-\frac{5}{x^2}$

24 $\left(\lim_{x\to 15} x - \lim_{x\to 15} 7\right)^{\frac{4}{3}}$

$(15-7)^{\frac{4}{3}}$

$= 16$

✓25 Graph

26 $\frac{1}{6}\cdot(8x+8)^3 + \left(1-\frac{1}{x^3}\right)^{-1}$

$\frac{d}{dx}\left[\frac{1}{6}\cdot(8x+8)^3\right] + \frac{d}{dx}\left[\left(1-\frac{1}{x^3}\right)^{-1}\right]$

$\frac{d}{dx}\left[\frac{1}{6}\cdot(8x+8)^3\right] + \frac{d}{dx}\left[\left(1-\frac{1}{x^3}\right)^{-1}\right]$

$\frac{d}{dx}\left[\frac{1}{6}\cdot(8x+8)^3\right] \quad \frac{d}{dx}\left[\left(1-\frac{1}{x^3}\right)^{-1}\right]$

$4(8x+8)^2 + \frac{d}{dx}\left[\left(1-\frac{1}{x^3}\right)^{-1}\right]$

$\frac{d}{dx}\left[\left(1-\frac{1}{x^3}\right)^{-1}\right]$

$4(8x+8)^2 - 3\left(1-\frac{1}{x^3}\right)^{-2} x^{-4}$

$4(8x+8)^2 - 3\frac{1}{\left(1-\frac{1}{x^3}\right)^2 x^4}$

$4x\ \frac{4\left(1-\frac{1}{x^3}\right)^2(8x+8)^2 - 3}{x^4\left(1-\frac{1}{x^3}\right)^2}$

✓27 Graph

32 Graph

33 $\lim_{x\to 6^-} (x+1)\left(\frac{|x+6|}{x+6}\right)$

$\left(\lim_{x\to 6^-} x + \lim_{x\to 6^-} 1\right) \cdot \left|\frac{\lim_{x\to 6^-} x + \lim_{x\to 6^-} 6}{\lim_{x\to 6^-} x + \lim_{x\to 6^-} 6}\right|$

$(6+1)\cdot\frac{|6+6|}{6+6}$

$=7$

34 $x^3+y^3=9$

$3x^2\frac{dx}{dt} + 3y^2\frac{dy}{dt}=0$

$\frac{dy}{dt} = -\frac{x^2}{y^2}\frac{dx}{dt}$

$\frac{dy}{dt} = -\left[\frac{1}{4}\right](-3)$

$\frac{dy}{dt} = \frac{3}{4}$

$\frac{dx}{dt}=-3$

$x=1, y=2$

35 $y=\cos x$, $x=\pi/2 \Rightarrow y=0$

$\left(\frac{\pi}{2}, 0\right)$ slope

$y' = \sin$ Slope (m) $= y'$ at $x = +\pi/2$

$= -\sin \pi/2 = -1$

$y-0=-1(x-\pi/2)$

$y=-x+\pi/2$