Benedict College
Department of CS, Physics and Engineering.
Math 241 Final Exam

Name____

Write your solution on the space providedor on a separet sheet of paper.

Evaluate the line integral along the curve C.

1) $\int_C (xz + y^2)\, ds$, C is the curve $\mathbf{r}(t) = (2 - 2t)\mathbf{i} + t\mathbf{j} - 2t\mathbf{k}$, $0 \le t \le 1$

Provide an appropriate response.

2) Find the direction in which the function is increasing most rapidly at the point P_0.

$f(x, y) = xy^2 - yx^2$, $P_0(2, -1)$

Solve the problem.

3) Let D be the region bounded below by the xy–plane, above by the sphere $x^2 + y^2 + z^2 = 100$, and on the sides by the cylinder $x^2 + y^2 = 25$. Set up the triple integral in cylindrical coordinates that gives the volume of D using the order of integration dz dr dθ.

Change the Cartesian integral to an equivalent polar integral, and then evaluate.

4) $\int_0^{11} \int_0^{\sqrt{121 - y^2}} (x^2 + y^2)\, dx\, dy$

Find the volume of the indicated region.

5) the region bounded above by the sphere $x^2 + y^2 + z^2 = 16$ and below by the cone $z = \sqrt{x^2 + y^2}$

Find the unit tangent vector of the given curve.

6) $\mathbf{r}(t) = (7 + 11t^8)\mathbf{i} + (10 + 2t^8)\mathbf{j} + (6 + 10t^8)\mathbf{k}$

Find all the second order partial derivatives of the given function.

7) $f(x, y) = \cos(xy^2)$

Evaluate the integral.

8) $\int_0^{\pi} \int_0^{\pi} \int_0^{6 \sin \phi} \rho^2 \sin \phi \, d\rho\, d\theta\, d\phi$

Find all the local maxima, local minima, and saddle points of the function.

9) $f(x, y) = x^3 + y^3 - 12x - 147y + 3$

1) - 1

2) $\left(\frac{5}{\sqrt{89}}\right)\mathrm{i} - \left(\frac{8}{\sqrt{89}}\right)\mathrm{j}$

3) $\int_0^{2\pi}\int_0^5\int_0^{\sqrt{100 - r^2}} r\,dz\,dr\,d\theta$

4) $\frac{14{,}641\pi}{8}$

5) $\frac{64}{3}\pi(2 - \sqrt{2})$

6) $\mathbf{T} = \frac{11}{15}\mathbf{i} + \frac{2}{15}\mathbf{j} + \frac{2}{3}\mathbf{k}$

7) $\frac{\partial^2 f}{\partial x^2} = -y^4 \cos xy^2;\ \frac{\partial^2 f}{\partial y^2} = -2x[2xy^2 \cos(xy^2) + \sin(xy^2)];\ \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial^2 f}{\partial x \partial y} =$
$-2y[xy^2 \cos(xy^2) + \sin(xy^2)]$;

8) $27\pi^2$

9) f(2, 7) = -699, local minimum; f(2, -7) = 673, saddle point; f(-2, 7) = -667, saddle point; f(-2, -7) = 705, local maximum